Eastern High School

AP Physics C Summer Assignment

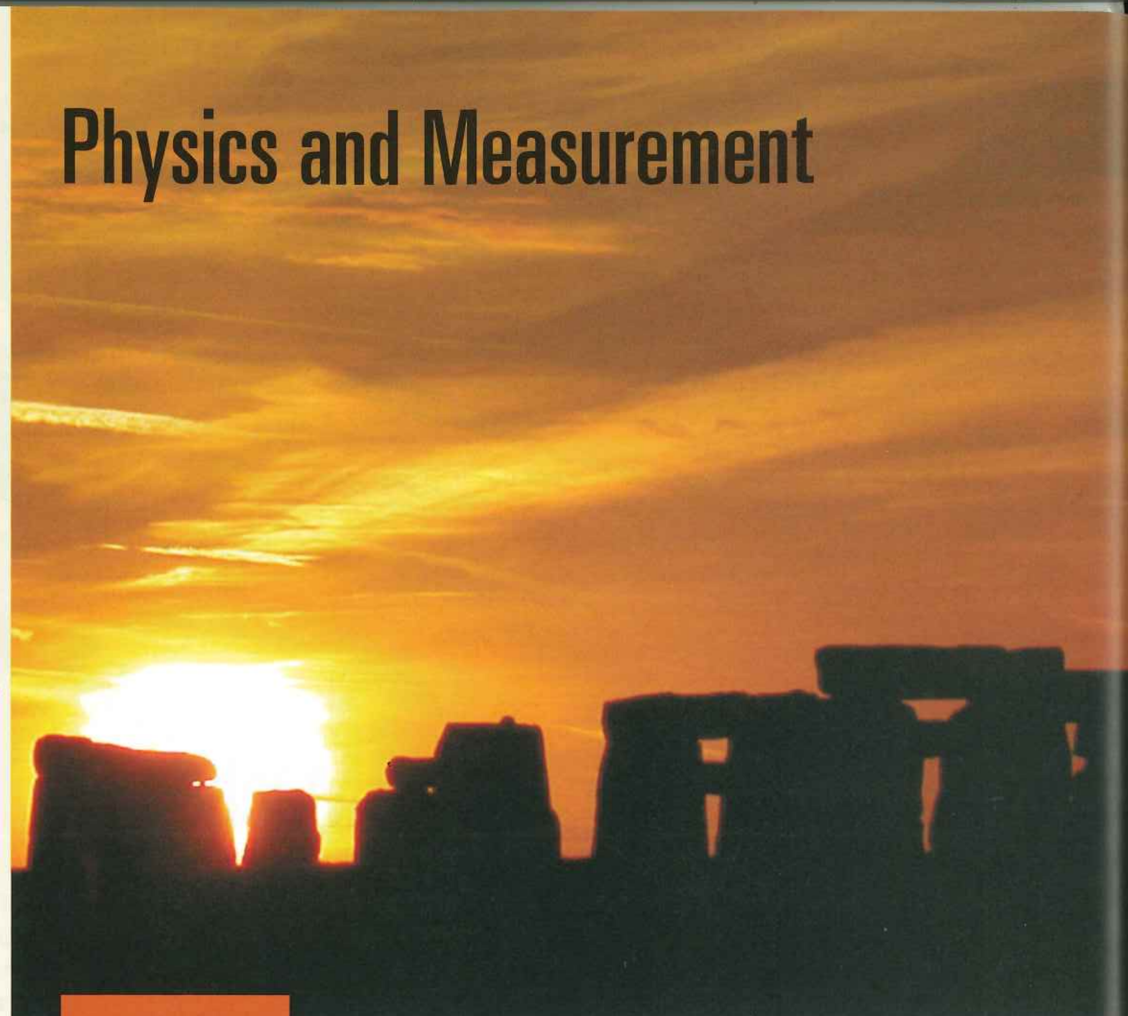
From Physics for Scientists and Engineers (10th ed.) by Serway, Jewett

All work is to be done using paper and pencil. Show complete answers. Show all work. Make your work neat and organized. You will turn in your work on the FIRST day of class.

- **Read Chapters 1 and 2**
- **Answer/Solve page 17-19 #'s 3, 11, 15, 16, 25, 27**
- **Answer/Solve page 48-51 #'s 2, 3, 4, 5, 7, 9, 11, 13, 14, 17, 23, 26, 28, 29, 32, 39**

Thank you. See you in September.

1 Physics and Measurement



Stonehenge, in southern England, was built thousands of years ago. Various hypotheses have been proposed about its function, including a burial ground, a healing site, and a place for ancestor worship. One of the more intriguing ideas suggests that Stonehenge was an observatory, allowing measurements of some of the quantities discussed in this chapter, such as position of objects in space and time intervals between repeating celestial events. *(Image copyright Stephen Inglis. Used under license from Shutterstock.com)*

STORYLINE **Each chapter in this textbook will begin with a paragraph** related to a storyline that runs throughout the text. The storyline centers on *you*: an inquisitive physics student. You could live anywhere in the world, but let's say you live in southern California, where one of the authors lives. Most of your observations will occur there, although you will take trips to other locations. As you go through your everyday activities, you see physics in action all around you. In fact, you can't get away from physics! As you observe phenomena at the beginning of each chapter, you will ask yourself, "Why does that happen?" You might take measurements with your smartphone. You might look for related videos on YouTube or photographs on an image search site. You are lucky indeed because, in addition to those resources, you have this textbook and the expertise of your instructor to help you understand the exciting physics surrounding you. Let's look at your first observations as we begin your storyline. You have just bought this textbook and have flipped through some of its pages. You notice a page of conversions on the inside back cover. You notice in the entries under "Length" the unit of a *light-year*. You say, "Wait a minute! (You will say this often in the upcoming chapters.) How can a unit based on a *year* be a unit of *length*?" As you look farther down the page, you see 1 kg ≈ 2.2 lb (lb is the abbreviation for *pound*; lb is from Latin *libra pondo*) under the heading "Some Approximations Useful for Estimation Problems." Noticing the "approximately equal" sign (≈), you wonder what the *exact* conversion is and look upward on the page to the heading "Mass," since a kilogram is a unit of mass. The relation between kilograms and pounds is not there! Why not? Your physics adventure has begun!

CONNECTIONS The second paragraph in each chapter will explain how the material in the chapter connects to that in the previous chapter and/or future

chapters. This feature will help you see that the textbook is not a collection of unrelated chapters, but rather is a structure of understanding that we are building, step by step. These paragraphs will provide a roadmap through the concepts and principles as they are introduced in the text. They will justify why the material in that chapter is presented at that time and help you to see the "big picture" of the study of physics. In this first chapter, of course, we cannot connect to a previous chapter. We will simply look ahead to the present chapter, in which we discuss some preliminary concepts of measurement, units, modeling, and estimation that we will need throughout *all* the chapters of the text.

1.1 Standards of Length, Mass, and Time

To describe natural phenomena, we must make measurements of various aspects of nature. Each measurement is associated with a physical quantity, such as the length of an object. The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout the book. In mechanics, the three fundamental quantities are *length, mass,* and *time.* All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. For example, if someone reports that a wall is 2 meters high and our standard unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Whatever is chosen as a standard must be readily accessible and must possess some property that can be measured reliably. Measurement standards used by different people in different places—throughout the Universe—must yield the same result. In addition, standards used for measurements must not change with time.

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI** (Système International), and its fundamental units of length, mass, and time are the *meter, kilogram,* and *second,* respectively. Other standards for SI fundamental units established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*).

Length

We can identify **length** as the distance between two points in space. In 1120, the king of England decreed that the standard of length in his country would be named the *yard* and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. Neither of these standards is constant in time; when a new king took the throne, length measurements changed! The French standard prevailed until 1799, when the legal standard of length in France became the **meter** (m), defined as one ten-millionth of the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris. Notice that this value is an Earth-based standard that does not satisfy the requirement that it can be used throughout the Universe.

Table 1.1 (page 4) lists approximate values of some measured lengths. You should study this table as well as the next two tables and begin to generate an intuition for what is meant by, for example, a length of 20 centimeters, a mass of 100 kilograms, or a time interval of 3.2×10^7 seconds.

As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. Current requirements of science and technology, however, necessitate more accuracy than that with which the separation between the lines on the bar can be determined. In the 1960s and 1970s, the meter was defined to be equal to

PITFALL PREVENTION 1.1
Reasonable Values Generating intuition about typical values of quantities when solving problems is important because you must think about your end result and determine if it seems reasonable. For example, if you are calculating the mass of a housefly and arrive at a value of 100 kg, this answer is *unreasonable* and there is an error somewhere.

TABLE 1.1 Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	2.7×10^{26}
Distance from the Earth to the most remote normal galaxies	3×10^{26}
Distance from the Earth to the nearest large galaxy (Andromeda)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One light-year	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

a

b

Jacques Brinon/AP Images

Focke Strangmann/AP Images

Figure 1.1 (a) International Prototype of the Kilogram, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) A cesium fountain atomic clock. The clock will neither gain nor lose a second in 20 million years.

1 650 763.73 wavelengths[1] of orange-red light emitted from a krypton-86 lamp. In October 1983, however, the meter was redefined as **the distance traveled by light in vacuum during a time interval of 1/299 792 458 second.** In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second. This definition of the meter is valid throughout the Universe based on our assumption that light is the same everywhere. The speed of light also allows us to define the **light-year**, as mentioned in the introductory storyline: the distance that light travels through empty space in one year. Use this definition and the speed of light to verify the length of a light-year in meters as given in Table 1.1.

Mass

We will find that the **mass** of an object is related to the amount of material that is present in the object, or to how much that object resists changes in its motion. Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it. The SI fundamental unit of mass, the **kilogram** (kg), is defined as **the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.** This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland (Fig. 1.1a). Table 1.2 lists approximate values of the masses of various objects.

In Chapter 5, we will discuss the difference between mass and weight. In anticipation of that discussion, let's look again at the approximate equivalence mentioned in the introductory storyline: 1 kg $\approx$ 2.2 lb. It would never be correct to claim that a number of kilograms *equals* a number of pounds, because these units represent different variables. A kilogram is a unit of *mass,* while a pound is a unit of *weight.* That's why an equality between kilograms and pounds is not given in the section of conversions for mass on the inside back cover of the textbook.

[1]We will use the standard international notation for numbers with more than three digits, in which groups of three digits are separated by spaces rather than commas. Therefore, 10 000 is the same as the common American notation of 10,000. Similarly, π = 3.14159265 is written as 3.141 592 65.

TABLE 1.2 Approximate Masses of Various Objects

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

TABLE 1.3 Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	4×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

Time

Before 1967, the standard of **time** was defined in terms of the *mean solar day.* (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The fundamental unit of a **second** (s) was defined as $(\frac{1}{60})(\frac{1}{60})(\frac{1}{24})$ of a mean solar day. This definition is based on the rotation of one planet, the Earth. Therefore, this motion does not provide a time standard that is universal.

In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an *atomic clock* (Fig. 1.1b), which measures vibrations of cesium atoms. One second is now defined as **9 192 631 770 times the period of vibration of radiation from the cesium-133 atom.**[2] Approximate values of time intervals are presented in Table 1.3.

You should note that we will use the notations *time* and *time interval* differently. A **time** is a description of an instant relative to a reference time. For example, t = 10.0 s refers to an instant 10.0 s after the instant we have identified as t = 0. As another example, a *time* of 11:30 a.m. means an instant 11.5 hours after our reference time of midnight. On the other hand, a **time interval** refers to *duration*: he required 30.0 minutes to finish the task. It is common to hear a "time of 30.0 minutes" in this latter example, but we will be careful to refer to measurements of duration as time intervals.

Units and Quantities In addition to SI, another system of units, the *U.S. customary system,* is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this book, we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of U.S. customary units in the study of classical mechanics.

In addition to the fundamental SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.4 (page 6). For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^3 m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is 10^3 grams (g), and 1 mega volt (MV) is 10^6 volts (V).

[2]Period is defined as the time interval needed for one complete vibration.

TABLE 1.4 Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{3}	kilo	k
10^{-21}	zepto	z	10^{6}	mega	M
10^{-18}	atto	a	10^{9}	giga	G
10^{-15}	femto	f	10^{12}	tera	T
10^{-12}	pico	p	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	E
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	c			
10^{-1}	deci	d			

The variables length, mass, and time are examples of *fundamental quantities.* Most other variables are *derived quantities,* those that can be expressed as a mathematical combination of fundamental quantities. Common examples are *area* (a product of two lengths) and *speed* (a ratio of a length to a time interval).

A table of the letters in the Greek alphabet is provided on the back endpaper of this book.

Another example of a derived quantity is **density.** The density ρ (Greek letter rho) of any substance is defined as its *mass per unit volume:*

$$\rho \equiv \frac{m}{V} \tag{1.1}$$

In terms of fundamental quantities, density is a ratio of a mass to a product of three lengths. Aluminum, for example, has a density of 2.70×10^3 kg/m^3, and iron has a density of 7.86×10^3 kg/m^3. An extreme difference in density can be imagined by thinking about holding a 10-centimeter (cm) cube of Styrofoam in one hand and a 10-cm cube of lead in the other. See Table 14.1 in Chapter 14 for densities of several materials.

QUICK QUIZ 1.1 In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? **(a)** The aluminum cam is larger. **(b)** The iron cam is larger. **(c)** Both cams have the same size.

1.2 Modeling and Alternative Representations

Most courses in general physics require the student to learn the skills of problem solving, and examinations usually include problems that test such skills. This section describes some useful ideas that will enable you to enhance your understanding of physical concepts, increase your accuracy in solving problems, eliminate initial panic or lack of direction in approaching a problem, and organize your work.

One of the primary problem-solving methods in physics is to form an appropriate **model** of the problem. **A model is a simplified substitute for the real problem that allows us to solve the problem in a relatively simple way.** As long as the predictions of the model agree to our satisfaction with the actual behavior of the real system, the model is valid. If the predictions do not agree, the model must be refined or replaced with another model. The power of modeling is in its ability to reduce a wide variety of very complex problems to a limited number of classes of problems that can be approached in similar ways.

In science, a model is very different from, for example, an architect's scale model of a proposed building, which appears as a smaller version of what it represents.

A scientific model is a theoretical construct and may have no visual similarity to the physical problem. A simple application of modeling is presented in Example 1.1, and we shall encounter many more examples of models as the text progresses.

Models are needed because the actual operation of the Universe is extremely complicated. Suppose, for example, we are asked to solve a problem about the Earth's motion around the Sun. The Earth is very complicated, with many processes occurring simultaneously. These processes include weather, seismic activity, and ocean movements as well as the multitude of processes involving human activity. Trying to maintain knowledge and understanding of all these processes is an impossible task.

The modeling approach recognizes that none of these processes affects the motion of the Earth around the Sun to a measurable degree. Therefore, these details are all ignored. In addition, as we shall find in Chapter 13, the size of the Earth does not affect the gravitational force between the Earth and the Sun; only the masses of the Earth and Sun and the distance between their centers determine this force. In a simplified model, the Earth is imagined to be a particle, an object with mass but zero size. This replacement of an extended object by a particle is called the **particle model,** which is used extensively in physics. By analyzing the motion of a particle with the mass of the Earth in orbit around the Sun, we find that the predictions of the particle's motion are in excellent agreement with the actual motion of the Earth.

The two primary conditions for using the particle model are as follows:

- The size of the actual object is of no consequence in the analysis of its motion.
- Any internal processes occurring in the object are of no consequence in the analysis of its motion.

Both of these conditions are in action in modeling the Earth as a particle. Its radius is not a factor in determining its motion, and internal processes such as thunderstorms, earthquakes, and manufacturing processes can be ignored.

Four categories of models used in this book will help us understand and solve physics problems. The first category is the **geometric model.** In this model, we form a geometric construction that represents the real situation. We then set aside the real problem and perform an analysis of the geometric construction. Consider a popular problem in elementary trigonometry, as in the following example.

Example 1.1 Finding the Height of a Tree

You wish to find the height of a tree but cannot measure it directly. You stand 50.0 m from the tree and determine that a line of sight from the ground to the top of the tree makes an angle of 25.0° with the ground. How tall is the tree?

SOLUTION

Figure 1.2 shows the tree and a right triangle corresponding to the information in the problem superimposed over it. (We assume that the tree is exactly perpendicular to a perfectly flat ground.) In the triangle, we know the length of the horizontal leg and the angle between the hypotenuse and the horizontal leg. We can find the height of the tree by calculating the length of the vertical leg. We do so with the tangent function:

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{h}{50.0\text{ m}}$$

$$h = (50.0\text{ m})\tan\theta = (50.0\text{ m})\tan 25.0° = 23.3\text{ m}$$

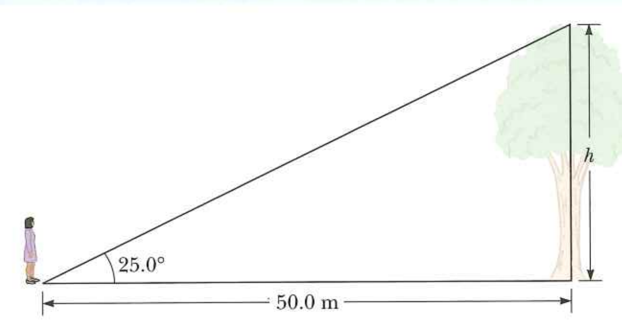


Figure 1.2 (Example 1.1) The height of a tree can be found by measuring the distance from the tree and the angle of sight to the top above the ground. This problem is a simple example of geometrically *modeling* the actual problem.

You may have solved a problem very similar to Example 1.1 but never thought about the notion of modeling. From the modeling approach, however, once we draw the triangle in Figure 1.2, the triangle is a geometric model of the real problem; it is a *substitute.* Until we reach the end of the problem, we do not imagine the problem to be about a *tree* but to be about a *triangle.* We use trigonometry to find the vertical leg of the triangle, leading to a value of 23.3 m. Because this leg *represents* the height of the tree, we can now return to the original problem and claim that the height of the tree is 23.3 m.

Other examples of geometric models include modeling the Earth as a perfect sphere, a pizza as a perfect disk, a meter stick as a long rod with no thickness, and an electric wire as a long, straight cylinder.

The particle model is an example of the second category of models, which we will call **simplification models.** In a simplification model, details that are not significant in determining the outcome of the problem are ignored. When we study rotation in Chapter 10, objects will be modeled as *rigid objects.* All the molecules in a rigid object maintain their exact positions with respect to one another. We adopt this simplification model because a spinning rock is much easier to analyze than a spinning block of gelatin, which is *not* a rigid object. Other simplification models will assume that quantities such as friction forces are negligible, remain constant, or are proportional to some power of the object's speed. We will assume *uniform* metal beams in Chapter 12, *laminar* flow of fluids in Chapter 14, *massless* springs in Chapter 15, *symmetric* distributions of electric charge in Chapter 23, *resistance-free* wires in Chapter 27, *thin* lenses in Chapter 34. These, and many more, are simplification models.

The third category is that of **analysis models,** which are general types of problems that we have solved before. An important technique in problem solving is to cast a new problem into a form similar to one we have already solved and which can be used as a model. As we shall see, there are about two dozen analysis models that can be used to solve most of the problems you will encounter. All of the analysis models in classical physics will be based on four simplification models: *particle, system, rigid object,* and *wave.* We will see our first analysis models in Chapter 2, where we will discuss them in more detail.

The fourth category of models is **structural models.** These models are generally used to understand the behavior of a system that is far different in scale from our macroscopic world—either much smaller or much larger—so that we cannot interact with it directly. As an example, the notion of a hydrogen atom as an electron in a circular orbit around a proton is a structural model of the atom. The ancient *geocentric* model of the Universe, in which the Earth is theorized to be at the center of the Universe, is an example of a structural model for something larger in scale than our macroscopic world.

Intimately related to the notion of modeling is that of forming **alternative representations** of the problem that you are solving. **A representation is a method of viewing or presenting the information related to the problem.** Scientists must be able to communicate complex ideas to individuals without scientific backgrounds. The best representation to use in conveying the information successfully will vary from one individual to the next. Some will be convinced by a well-drawn graph, and others will require a picture. Physicists are often persuaded to agree with a point of view by examining an equation, but nonphysicists may not be convinced by this mathematical representation of the information.

A word problem, such as those at the ends of the chapters in this book, is one representation of a problem. In the "real world" that you will enter after graduation, the initial representation of a problem may be just an existing situation, such as the effects of climate change or a patient in danger of dying. You may have to identify the important data and information, and then cast the situation yourself into an equivalent word problem!

Considering alternative representations can help you think about the information in the problem in several different ways to help you understand and solve it. Several types of representations can be of assistance in this endeavor:

- **Mental representation.** From the description of the problem, imagine a scene that describes what is happening in the word problem, then let time progress so that you understand the situation and can predict what changes will occur in the situation. This step is critical in approaching *every* problem.
- **Pictorial representation.** Drawing a picture of the situation described in the word problem can be of great assistance in understanding the problem. In Example 1.1, the pictorial representation in Figure 1.2 allows us to identify the triangle as a geometric model of the problem. In architecture, a blueprint is a pictorial representation of a proposed building.

  Generally, a pictorial representation describes *what you would see* if you were observing the situation in the problem. For example, Figure 1.3 shows a pictorial representation of a baseball player hitting a short pop foul. Any coordinate axes included in your pictorial representation will be in two dimensions: *x* and *y* axes.
- **Simplified pictorial representation.** It is often useful to redraw the pictorial representation without complicating details by applying a simplification model. This process is similar to the discussion of the particle model described earlier. In a pictorial representation of the Earth in orbit around the Sun, you might draw the Earth and the Sun as spheres, with possibly some attempt to draw continents to identify which sphere is the Earth. In the simplified pictorial representation, the Earth and the Sun would be drawn simply as dots, representing particles, with appropriate labels. Figure 1.4 shows a simplified pictorial representation corresponding to the pictorial representation of the baseball trajectory in Figure 1.3. The notations v_x and v_y refer to the components of the velocity vector for the baseball. We will study vector components in Chapter 3. We shall use such simplified pictorial representations throughout the book.
- **Graphical representation.** In some problems, drawing a graph that describes the situation can be very helpful. In mechanics, for example, position–time graphs can be of great assistance. Similarly, in thermodynamics, pressure–volume graphs are essential to understanding. Figure 1.5 shows a graphical representation of the position as a function of time of a block on the end of a vertical spring as it oscillates up and down. Such a graph is helpful for understanding simple harmonic motion, which we study in Chapter 15.

  A graphical representation is different from a pictorial representation, which is also a two-dimensional display of information but whose axes, if any, represent *length* coordinates. In a graphical representation, the axes may represent *any* two related variables. For example, a graphical representation may have axes for temperature and time. The graph in Figure 1.5 has axes of vertical position *y* and time *t*. Therefore, in comparison to a pictorial representation, a graphical representation is generally *not* something you would see when observing the situation in the problem with your eyes.
- **Tabular representation.** It is sometimes helpful to organize the information in tabular form to help make it clearer. For example, some students find that making tables of known quantities and unknown quantities is helpful. The periodic table of the elements is an extremely useful tabular representation of information in chemistry and physics.
- **Mathematical representation.** The ultimate goal in solving a problem is often the mathematical representation. You want to move from the information contained in the word problem, through various representations of the problem that allow you to understand what is happening, to one or more equations that represent the situation in the problem and that can be solved mathematically for the desired result.

Figure 1.3 A pictorial representation of a pop foul being hit by a baseball player.

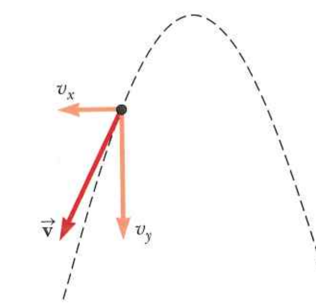


Figure 1.4 A simplified pictorial representation for the situation shown in Figure 1.3.

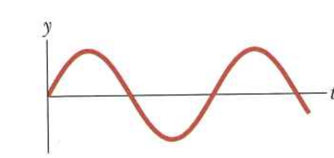


Figure 1.5 A graphical representation of the position as a function of time of a block hanging from a spring and oscillating.

1.3 Dimensional Analysis

In physics, the word *dimension* denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet, meters, or furlongs, which are all different units for expressing the dimension of length.

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively.[3] We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation, the dimensions of speed are written $[v] = \mathrm{L/T}$. As another example, the dimensions of area A are $[A] = \mathrm{L}^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.5. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

In many situations, you may have to check a specific equation to see if it matches your expectations. A useful procedure for doing that, called **dimensional analysis,** can be used because dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to determine whether an expression has the correct form. Any relationship can be correct only if the dimensions on both sides of the equation are the same.

PITFALL PREVENTION 1.2
Symbols for Quantities Some quantities have a small number of symbols that represent them. For example, the symbol for time is almost always t. Other quantities might have various symbols depending on the usage. Length may be described with symbols such as x, y, and z (for position); r (for radius); a, b, and c (for the legs of a right triangle); ℓ (for the length of an object); d (for a distance); h (for a height); and so forth.

To illustrate this procedure, suppose you are interested in an equation for the position x of a car at a time t if the car starts from rest at $x = 0$ and moves with constant acceleration a. The correct expression for this situation is $x = \frac{1}{2}at^2$ as we show in Chapter 2. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, $\mathrm{L/T^2}$ (Table 1.5), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$\mathrm{L} = \frac{\mathrm{L}}{\cancel{\mathrm{T}^2}} \cdot \cancel{\mathrm{T}^2} = \mathrm{L}$$

The dimensions of time cancel as shown, leaving the dimension of length on the right-hand side to match that on the left.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = \mathrm{L} = \mathrm{L}^1\mathrm{T}^0$$

TABLE 1.5 Dimensions and Units of Four Derived Quantities

Quantity	Area (A)	Volume (V)	Speed (v)	Acceleration (a)
Dimensions	L^2	L^3	L/T	$\mathrm{L/T^2}$
SI units	m^2	m^3	m/s	$\mathrm{m/s^2}$
U.S. customary units	ft^2	ft^3	ft/s	$\mathrm{ft/s^2}$

[3]The *dimensions* of a quantity will be symbolized by a capitalized, nonitalic letter such as L or T. The *algebraic symbol* for the quantity itself will be an italicized letter such as L for the length of an object or t for time.

Because the dimensions of acceleration are $\mathrm{L/T^2}$ and the dimension of time is T, we have

$$(\mathrm{L/T^2})^n\mathrm{T}^m = \mathrm{L}^1\mathrm{T}^0 \rightarrow (\mathrm{L}^n\mathrm{T}^{m-2n}) = \mathrm{L}^1\mathrm{T}^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for n, gives us $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$.

QUICK QUIZ 1.2 True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

Example 1.2 Analysis of an Equation

Show that the expression $v = at$, where v represents speed, a acceleration, and t an instant of time, is dimensionally correct.

SOLUTION

Identify the dimensions of v from Table 1.5: $[v] = \dfrac{\mathrm{L}}{\mathrm{T}}$

Identify the dimensions of a from Table 1.5 and multiply by the dimensions of t: $[at] = \dfrac{\mathrm{L}}{\mathrm{T}^{\cancel{2}}}\,\cancel{\mathrm{T}} = \dfrac{\mathrm{L}}{\mathrm{T}}$

Therefore, $v = at$ is dimensionally correct because we have the same dimensions on both sides. (If the expression were given as $v = at^2$, it would be dimensionally *incorrect.* Try it and see!)

Example 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

SOLUTION

Write an expression for a with a dimensionless constant of proportionality k: $a = kr^nv^m$

Substitute the dimensions of a, r, and v: $\dfrac{\mathrm{L}}{\mathrm{T}^2} = \mathrm{L}^n\left(\dfrac{\mathrm{L}}{\mathrm{T}}\right)^m = \dfrac{\mathrm{L}^{n+m}}{\mathrm{T}^m}$

Equate the exponents of L and T so that the dimensional equation is balanced: $n + m = 1$ and $m = 2$

Solve the two equations for n: $n = -1$

Write the acceleration expression: $a = kr^{-1}v^2 = k\dfrac{v^2}{r}$

In Section 4.4 on uniform circular motion, we show that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in $\mathrm{m/s^2}$.

PITFALL PREVENTION 1.3

Always Include Units When performing calculations with numerical values, include the units for every quantity and carry the units through the entire calculation. Avoid the temptation to drop the units early and then attach the expected units once you have an answer. By including the units in every step, you can detect errors if the units for the answer turn out to be incorrect.

1.4 Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another or convert within a system (for example, from kilometers to meters). Conversion factors between SI and U.S. customary units of length are as follows:

$$1 \text{ mile} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Like dimensions, units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0 \cancel{\text{in.}})\left(\frac{2.54 \text{ cm}}{1 \cancel{\text{in.}}}\right) = 38.1 \text{ cm}$$

where the ratio in parentheses is equal to 1. We express 1 as 2.54 cm/1 in. (rather than 1 in./2.54 cm) so that the unit "inch" in the denominator cancels with the unit in the original quantity. The remaining unit is the centimeter, our desired result.

QUICK QUIZ 1.3 The distance between two cities is 100 mi. What is the number of kilometers between the two cities? **(a)** smaller than 100 **(b)** larger than 100 **(c)** equal to 100

Example 1.4 Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is the driver exceeding the speed limit of 75.0 mi/h?

SOLUTION

Convert meters to miles and seconds to hours:

$$(38.0 \cancel{\text{m}}/\cancel{\text{s}})\left(\frac{1 \text{ mi}}{1\,609 \cancel{\text{m}}}\right)\left(\frac{60 \cancel{\text{s}}}{1 \cancel{\text{min}}}\right)\left(\frac{60 \cancel{\text{min}}}{1 \text{ h}}\right) = 85.0 \text{ mi/h}$$

The driver is indeed exceeding the speed limit and should slow down.

WHAT IF? What if the driver were from outside the United States and is familiar with speeds measured in kilometers per hour? What is the speed of the car in km/h?

Answer We can convert our final answer to the appropriate units:

$$(85.0 \cancel{\text{mi}}/\text{h})\left(\frac{1.609 \text{ km}}{1 \cancel{\text{mi}}}\right) = 137 \text{ km/h}$$

Figure 1.6 shows an automobile speedometer displaying speeds in both mi/h and km/h. Can you check the conversion we just performed using this photograph?

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Figure 1.6 The speedometer of a vehicle that shows speeds in both miles per hour and kilometers per hour.

1.5 Estimates and Order-of-Magnitude Calculations

Suppose someone asks you the number of bits of data on a typical Blu-ray Disc. In response, it is not generally expected that you would provide the exact number but rather an estimate, which may be expressed in scientific notation. The estimate may be made even more approximate by expressing it as an **order of magnitude**, which is a power of 10 determined as follows:

1. Express the number in scientific notation, with the multiplier of the power of 10 between 1 and 10 and a unit.
2. If the multiplier is less than 3.162 (the square root of 10), the order of magnitude of the number is the power of 10 in the scientific notation. If the multiplier is greater than 3.162, the order of magnitude is one larger than the power of 10 in the scientific notation.

We use the symbol ~ for "is on the order of." Use the procedure above to verify the orders of magnitude for the following lengths:

$$0.008\,6 \text{ m} \sim 10^{-2} \text{ m} \qquad 0.002\,1 \text{ m} \sim 10^{-3} \text{ m} \qquad 720 \text{ m} \sim 10^{3} \text{ m}$$

Usually, when an order-of-magnitude estimate is made, the results are reliable to within about a factor of 10.

Inaccuracies caused by guessing too low for one number are often canceled by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work because you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head or with minimal mathematical manipulation on paper. Because of the simplicity of these types of calculations, they can be performed on a small scrap of paper and are often called *back-of-the-envelope calculations.*

Example 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average human lifetime.

SOLUTION

We start by guessing that the typical human lifetime is about 70 years. Think about the average number of breaths that a person takes in 1 min. This number varies depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate. (This estimate is certainly closer to the true average value than an estimate of 1 breath per minute or 100 breaths per minute.)

Find the approximate number of minutes in a year:

$$1 \cancel{\text{yr}}\left(\frac{400 \cancel{\text{days}}}{1 \cancel{\text{yr}}}\right)\left(\frac{25 \cancel{\text{h}}}{1 \cancel{\text{day}}}\right)\left(\frac{60 \text{ min}}{1 \cancel{\text{h}}}\right) = 6 \times 10^5 \text{ min}$$

Find the approximate number of minutes in a 70-year lifetime:

$$\text{number of minutes} = (70 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 4 \times 10^7 \text{ min}$$

Find the approximate number of breaths in a lifetime:

$$\text{number of breaths} = (10 \text{ breaths/min})(4 \times 10^7 \text{ min}) = 4 \times 10^8 \text{ breaths}$$

Therefore, a person takes on the order of 10^9 breaths in a lifetime. Notice how much simpler it is in the first calculation above to multiply 400 × 25 than it is to work with the more accurate 365 × 24.

WHAT IF? What if the average lifetime were estimated as 80 years instead of 70? Would that change our final estimate?

Answer We could claim that (80 yr)(6×10^5 min/yr) = 5×10^7 min, so our final estimate should be 5×10^8 breaths. This answer is still on the order of 10^9 breaths, so an order-of-magnitude estimate would be unchanged.

1.6 Significant Figures

When certain quantities are measured, the measured values are known only to within the limits of the experimental uncertainty. The value of this uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experimenter, and the number of measurements performed. The number of

significant figures in a measurement can be used to express something about the uncertainty. The number of significant figures is related to the number of numerical digits used to express the measurement, as we discuss below.

As an example of significant figures, suppose we are asked to measure the radius of a Blu-ray Disc using a meterstick as a measuring instrument. Let us assume the accuracy to which we can measure the radius of the disc is ±0.1 cm. Because of the uncertainty of ±0.1 cm, if the radius is measured to be 6.0 cm, we can claim only that its radius lies somewhere between 5.9 cm and 6.1 cm. In this case, we say that the measured value of 6.0 cm has two significant figures. Note that *the significant figures include the first estimated digit.* Therefore, we could write the radius as (6.0 ± 0.1) cm.

Zeros may or may not be significant figures. Those used to position the decimal point in such numbers as 0.03 and 0.007 5 are not significant. Therefore, there are one and two significant figures, respectively, in these two values. When the zeros come after other digits, however, there is the possibility of misinterpretation. For example, suppose the mass of an object is given as 1 500 g. This value is ambiguous because we do not know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement. To remove this ambiguity, it is common to use scientific notation to indicate the number of significant figures. In this case, we would express the mass as 1.5×10^3 g if there are two significant figures in the measured value, 1.50×10^3 g if there are three significant figures, and 1.500×10^3 g if there are four. The same rule holds for numbers less than 1, so 2.3×10^{-4} has two significant figures (and therefore could be written 0.000 23) and 2.30×10^{-4} has three significant figures (and therefore written as 0.000 230).

In problem solving, we often combine quantities mathematically through multiplication, division, addition, subtraction, and so forth. When doing so, you must make sure that the result has the appropriate number of significant figures. A good rule of thumb to use in determining the number of significant figures that can be claimed in a multiplication or a division is as follows:

> When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures. The same rule applies to division.

Let's apply this rule to find the area of the Blu-ray Disc whose radius we measured above. Using the equation for the area of a circle,

$$A = \pi r^2 = \pi(6.0 \text{ cm})^2 = 1.1 \times 10^2 \text{ cm}^2$$

If you perform this calculation on your calculator, you will likely see 113.097 335 5. It should be clear that you don't want to keep all of these digits, but you might be tempted to report the result as 113 cm². This result is not justified because it has three significant figures, whereas the radius only has two. Therefore, we must report the result with only two significant figures as shown above.

For addition and subtraction, you must consider the number of decimal places when you are determining how many significant figures to report:

> When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum or difference.

PITFALL PREVENTION 1.4

Read Carefully Notice that the rule for addition and subtraction is different from that for multiplication and division. For addition and subtraction, the important consideration is the number of *decimal places,* not the number of *significant figures.*

As an example of this rule, consider the sum

$$23.2 + 5.174 = 28.4$$

Notice that we do not report the answer as 28.374 because the lowest number of decimal places is one, for 23.2. Therefore, our answer must have only one decimal place.

The rule for addition and subtraction can often result in answers that have a different number of significant figures than the quantities with which you start. For example, consider these operations that satisfy the rule:

$$1.000\,1 + 0.000\,3 = 1.000\,4$$
$$1.002 - 0.998 = 0.004$$

In the first example, the result has five significant figures even though one of the terms, 0.000 3, has only one significant figure. Similarly, in the second calculation, the result has only one significant figure even though the numbers being subtracted have four and three, respectively.

> In this book, most of the numerical examples and end-of-chapter problems will yield answers having three significant figures. When carrying out estimation calculations, we shall typically work with a single significant figure.

◀ Significant figure guidelines used in this book

If the number of significant figures in the result of a calculation must be reduced, there is a general rule for rounding numbers: the last digit retained is increased by 1 if the last digit dropped is greater than 5. (For example, 1.346 becomes 1.35.) If the last digit dropped is less than 5, the last digit retained remains as it is. (For example, 1.343 becomes 1.34.) If the last digit dropped is equal to 5, the remaining digit should be rounded to the nearest even number. (This rule helps avoid accumulation of errors in long arithmetic processes.)

In a long calculation involving multiple steps, it is very important to delay the rounding of numbers until you have the final result, in order to avoid error accumulation. Wait until you are ready to copy the final answer from your calculator before rounding to the correct number of significant figures. In this book, we display numerical values rounded off to two or three significant figures. This occasionally makes some mathematical manipulations look odd or incorrect. For instance, looking ahead to Example 3.5 on page 62, you will see the operation −17.7 km + 34.6 km = 17.0 km. This looks like an incorrect subtraction, but that is only because we have rounded the numbers 17.7 km and 34.6 km for display. If all digits in these two intermediate numbers are retained and the rounding is only performed on the final number, the correct three-digit result of 17.0 km is obtained.

PITFALL PREVENTION 1.5

Symbolic Solutions When solving problems, it is very useful to perform the solution completely in algebraic form and wait until the very end to enter numerical values into the final symbolic expression. This method will save many calculator keystrokes, especially if some quantities cancel so that you never have to enter their values into your calculator! In addition, you will only need to round once, on the final result.

Example 1.6 Installing a Carpet

A carpet is to be installed in a rectangular room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

SOLUTION

If you multiply 12.71 m by 3.46 m on your calculator, you will see an answer of 43.976 6 m². How many of these numbers should you claim? Our rule of thumb for multiplication tells us that you can claim only the number of significant figures in your answer as are present in the measured quantity having the lowest number of significant figures. In this example, the lowest number of significant figures is three in 3.46 m, so we should express our final answer as 44.0 m².

Summary

Definitions

The three fundamental physical quantities of mechanics are **length**, **mass**, and **time**, which in the SI system have the units **meter** (m), **kilogram** (kg), and **second** (s), respectively. These fundamental quantities cannot be defined in terms of more basic quantities.

The **density** of a substance is defined as its *mass per unit volume:*

$$\rho \equiv \frac{m}{V} \tag{1.1}$$

continued

Concepts and Principles

The method of **dimensional analysis** is very powerful in solving physics problems. Dimensions can be treated as algebraic quantities. By making estimates and performing order-of-magnitude calculations, you should be able to approximate the answer to a problem when there is not enough information available to specify an exact solution completely.

Problem-solving skills and physical understanding can be improved by **modeling** the problem and by constructing **alternative representations** of the problem. Models helpful in solving problems include **geometric, simplification, analysis,** and **structural models.** Helpful representations include the **mental, pictorial, simplified pictorial, graphical, tabular,** and **mathematical representations.**

When you compute a result from several measured numbers, each of which has a certain accuracy, you should give the result with the correct number of **significant figures.**

When **multiplying** several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures. The same rule applies to **division.**

When numbers are **added** or **subtracted,** the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum or difference.

Think–Pair–Share

See the Preface for an explanation of the icons used in this problems set. For additional assessment items for this section, go to WEBASSIGN

1. A student is supplied with a stack of copy paper, ruler, compass, scissors, and a sensitive balance. He cuts out various shapes in various sizes, calculates their areas, measures their masses, and prepares the graph of Figure TP1.1. (a) Consider the fourth experimental point from the top. How far is it vertically from the best-fit straight line? Express your answer as a difference in vertical-axis coordinate. (b) Express your answer as a percentage. (c) Calculate the slope of the line. (d) State what the graph demonstrates, referring to the shape of the graph and the results of parts (b) and (c). (e) Describe whether this result should be expected theoretically. (f) Describe the physical meaning of the slope.

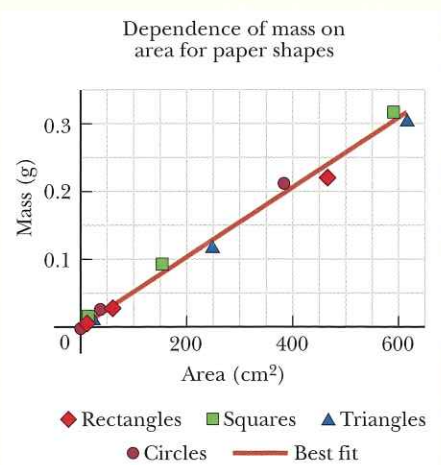


Figure TP1.1

2. ACTIVITY Have each person in the group measure the height of another person using a meter stick with metric distances on one side and U.S. customary distances, such as inches, on the other side. Record the height to the nearest centimeter and to the nearest half-inch. For each person, divide his or her height in centimeters by the height in inches. Compare the results of this division for everyone in your group. What can you say about the results?

3. ACTIVITY Gather together a number of U.S. pennies, either from your instructor or from the members of your group. Divide up the pennies into two samples: (1) those with dates of 1981 or earlier, and (2) those with dates of 1983 and later (exclude 1982 pennies from your sample). Find the total mass of all the pennies in each sample. Then divide each of these total masses by the number of pennies in its corresponding sample, to find the average penny mass in each sample. Discuss why the results are different for the two samples.

4. ACTIVITY Discuss in your group the process by which you can obtain the best measurement of the thickness of a single sheet of paper in Chapters 1–5 of this book. Perform that measurement and express it with an appropriate number of significant figures and uncertainty. From that measurement, predict the total thickness of the pages in Volume 1 of this book (Chapters 1–21). After making your prediction, measure the thickness of Volume 1. Is your measurement within the range of your prediction and its associated uncertainty?

Problems

See the Preface for an explanation of the icons used in this problems set. For additional assessment items for this section, go to WEBASSIGN

Note: Consult the endpapers, appendices, and tables in the text whenever necessary in solving problems. For this chapter, Table 14.1 and Appendix B.3 may be particularly useful. Answers to odd-numbered problems appear in the back of the book.

SECTION 1.1 Standards of Length, Mass, and Time

1. QIC (a) Use information on the endpapers of this book to calculate the average density of the Earth. (b) Where does the value fit among those listed in Table 14.1 in Chapter 14? Look up the density of a typical surface rock like granite in another source and compare it with the density of the Earth.

2. QIC A proton, which is the nucleus of a hydrogen atom, can be modeled as a sphere with a diameter of 2.4 fm and a mass of 1.67×10^{-27} kg. (a) Determine the density of the proton. (b) State how your answer to part (a) compares with the density of osmium, given in Table 14.1 in Chapter 14.

3. V Two spheres are cut from a certain uniform rock. One has radius 4.50 cm. The mass of the other is five times greater. Find its radius.

4. S What mass of a material with density ρ is required to make a hollow spherical shell having inner radius r_1 and outer radius r_2?

5. CR You have been hired by the defense attorney as an expert witness in a lawsuit. The plaintiff is someone who just returned from being a passenger on the first orbital space tourist flight. Based on a travel brochure offered by the space travel company, the plaintiff expected to be able to see the Great Wall of China from his orbital height of 200 km above the Earth's surface. He was unable to do so, and is now demanding that his fare be refunded and to receive additional financial compensation to cover his great disappointment. Construct the basis for an argument for the defense that shows that his expectation of seeing the Great Wall from orbit was unreasonable. The Wall is 7 m wide at its widest point and the normal visual acuity of the human eye is 3×10^{-4} rad. (Visual acuity is the smallest subtended angle that an object can make at the eye and still be recognized; the subtended angle in radians is the ratio of the width of an object to the distance of the object from your eyes.)

SECTION 1.2 Modeling and Alternative Representations

6. A surveyor measures the distance across a straight river by the following method (Fig. P1.6). Starting directly across from a tree on the opposite bank, she walks d = 100 m along the riverbank to establish a baseline. Then she sights across to the tree. The angle from her baseline to the tree is $\theta = 35.0°$. How wide is the river?

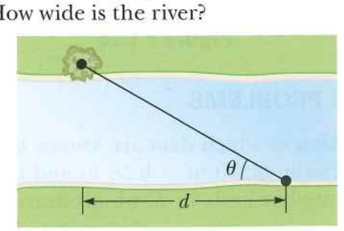


Figure P1.6

7. A crystalline solid consists of atoms stacked up in a repeating lattice structure. Consider a crystal as shown in Figure P1.7a. The atoms reside at the corners of cubes of side L = 0.200 nm. One piece of evidence for the regular arrangement of atoms comes from the flat surfaces along which a crystal separates, or cleaves, when it is broken. Suppose this crystal cleaves along a face diagonal as shown in Figure P1.7b. Calculate the spacing d between two adjacent atomic planes that separate when the crystal cleaves.

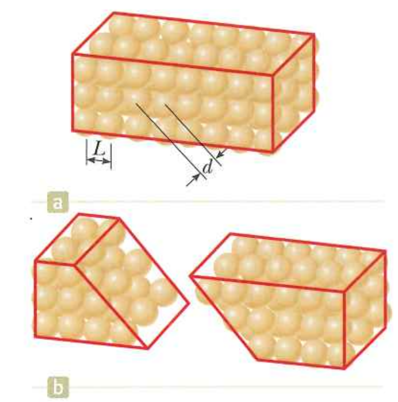


Figure P1.7

SECTION 1.3 Dimensional Analysis

8. The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position as $x = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?

9. Which of the following equations are dimensionally correct? (a) $v_f = v_i + ax$ (b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$

10. (a) Assume the equation $x = At^3 + Bt$ describes the motion of a particular object, with x having the dimension of length and t having the dimension of time. Determine the dimensions of the constants A and B. (b) Determine the dimensions of the derivative $dx/dt = 3At^2 + B$.

SECTION 1.4 Conversion of Units

11. V A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kilograms per cubic meter).

12. *Why is the following situation impossible?* A student's dormitory room measures 3.8 m by 3.6 m, and its ceiling is 2.5 m high. After the student completes his physics course, he displays his dedication by completely wallpapering the walls of the room with the pages from his copy of volume 1 (Chapters 1–21) of this textbook. He even covers the door and window.

13. T One cubic meter (1.00 m^3) of aluminum has a mass of 2.70×10^3 kg, and the same volume of iron has a mass of 7.86×10^3 kg. Find the radius of a solid aluminum sphere that will balance a solid iron sphere of radius 2.00 cm on an equal-arm balance.

14. S Let ρ_{Al} represent the density of aluminum and ρ_{Fe} that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius r_{Fe} on an equal-arm balance.

15. T One gallon of paint (volume $= 3.78 \times 10^{-3}$ m^3) covers an area of 25.0 m^2. What is the thickness of the fresh paint on the wall?

16. V An auditorium measures 40.0 m $\times$ 20.0 m $\times$ 12.0 m. The density of air is 1.20 kg/m^3. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?

SECTION 1.5 Estimates and Order-of-Magnitude Calculations

Note: In your solutions to Problems 17 and 18, state the quantities you measure or estimate and the values you take for them.

17. (a) Compute the order of magnitude of the mass of a bathtub half full of water. (b) Compute the order of magnitude of the mass of a bathtub half full of copper coins.

18. To an order of magnitude, how many piano tuners reside in New York City? The physicist Enrico Fermi was famous for asking questions like this one on oral Ph.D. qualifying examinations.

19. CR Your roommate is playing a video game from the latest *Star Wars* movie while you are studying physics. Distracted by the noise, you go to see what is on the screen. The game involves trying to fly a spacecraft through a crowded field of asteroids in the asteroid belt around the Sun. You say to him, "Do you know that the game you are playing is very unrealistic? The asteroid belt is not that crowded and you don't have to maneuver through it like that!" Distracted by your statement, he accidentally allows his spacecraft to strike an asteroid, just missing the high score. He turns to you in disgust and says, "Yeah, prove it." You say, "Okay, I've learned recently that the highest concentration of asteroids is in a doughnut-shaped region between the Kirkwood gaps at radii of 2.06 AU and 3.27 AU from the Sun. There are an estimated 10^9 asteroids of radius 100 m or larger, like those in your video game, in this region . . ." Finish your argument with a calculation to show that the number of asteroids in the space near a spacecraft is tiny. (An astronomical unit—AU—is the mean distance of the Earth from the Sun: 1 AU $= 1.496 \times 10^{11}$ m.)

SECTION 1.6 Significant Figures

Note: Appendix B.8 on propagation of uncertainty may be useful in solving some problems in this section.

20. V How many significant figures are in the following numbers? (a) 78.9 ± 0.2 (b) 3.788×10^9 (c) 2.46×10^{-6} (d) 0.005 3

21. The *tropical year,* the time interval from one vernal equinox to the next vernal equinox, is the basis for our calendar. It contains 365.242 199 days. Find the number of seconds in a tropical year.

Note: The next seven problems call on mathematical skills from your prior education that will be useful throughout this course.

22. **Review.** The average density of the planet Uranus is 1.27×10^3 kg/m^3. The ratio of the mass of Neptune to that of Uranus is 1.19. The ratio of the radius of Neptune to that of Uranus is 0.969. Find the average density of Neptune.

23. **Review.** In a community college parking lot, the number of ordinary cars is larger than the number of sport utility vehicles by 94.7%. The difference between the number of cars and the number of SUVs is 18. Find the number of SUVs in the lot.

24. **Review.** Find every angle θ between 0 and 360° for which the ratio of sin θ to cos θ is -3.00.

25. **Review.** The ratio of the number of sparrows visiting a bird feeder to the number of more interesting birds is 2.25. On a morning when altogether 91 birds visit the feeder, what is the number of sparrows?

26. **Review.** Prove that one solution of the equation

$$2.00x^4 - 3.00x^3 + 5.00x = 70.0$$

is $x = -2.22$.

27. S **Review.** From the set of equations

$$p = 3q$$
$$pr = qs$$
$$\tfrac{1}{2}pr^2 + \tfrac{1}{2}qs^2 = \tfrac{1}{2}qt^2$$

involving the unknowns p, q, r, s, and t, find the value of the ratio of t to r.

28. Q|C S **Review.** Figure P1.28 shows students studying the thermal conduction of energy into cylindrical blocks of ice. As we will see in Chapter 19, this process is described by the equation

$$\frac{Q}{\Delta t} = \frac{k\pi d^2(T_h - T_c)}{4L}$$

For experimental control, in one set of trials all quantities except d and Δt are constant. (a) If d is made three times larger, does the equation predict that Δt will get larger or get smaller? By what factor? (b) What pattern of proportionality of Δt to d does the equation predict? (c) To display this proportionality as a straight line on a graph, what quantities should you plot on the horizontal and vertical axes? (d) What expression represents the theoretical slope of this graph?

Figure P1.28

ADDITIONAL PROBLEMS

29. In a situation in which data are known to three significant digits, we write 6.379 m = 6.38 m and 6.374 m = 6.37 m. When a number ends in 5, we arbitrarily choose to write 6.375 m = 6.38 m. We could equally well write 6.375 m = 6.37 m, "rounding down" instead of "rounding up," because we would change the number 6.375 by equal increments in both cases. Now consider an order-of-magnitude estimate, in which factors of change rather than increments are important. We write 500 m $\sim 10^3$ m because 500 differs from 100 by a factor of 5 while it differs from 1 000 by only a factor of 2. We write 437 m $\sim 10^3$ m and 305 m $\sim 10^2$ m. What distance differs from 100 m and from 1 000 m by equal factors so that we could equally well choose to represent its order of magnitude as $\sim 10^2$ m or as $\sim 10^3$ m?

30. BIO Q|C (a) What is the order of magnitude of the number of micro organisms in the human intestinal tract? A typical bacterial length scale is 10^{-6} m. Estimate the intestinal volume and assume 1% of it is occupied by bacteria. (b) Does the number of bacteria suggest whether the bacteria are beneficial, dangerous, or neutral for the human body? What functions could they serve?

31. The distance from the Sun to the nearest star is about 4×10^{16} m. The Milky Way galaxy (Fig. P1.31) is roughly a disk of diameter 10^{21} m and thickness $\sim 10^{19}$ m. Find the order of magnitude of the number of stars in the Milky Way. Assume the distance between the Sun and our nearest neighbor is typical.

Figure P1.31 The Milky Way galaxy.

32. *Why is the following situation impossible?* In an effort to boost interest in a television game show, each weekly winner is offered an additional \$1 million bonus prize if he or she can personally count out that exact amount from a supply of one-dollar bills. The winner must do this task under supervision by television show executives and within one 40-hour work week. To the dismay of the show's producers, most contestants succeed at the challenge.

33. BIO Bacteria and other prokaryotes are found deep underground, in water, and in the air. One micron (10^{-6} m) is a typical length scale associated with these microbes. (a) Estimate the total number of bacteria and other prokaryotes on the Earth. (b) Estimate the total mass of all such microbes.

34. Q|C A spherical shell has an outside radius of 2.60 cm and an inside radius of a. The shell wall has uniform thickness and is made of a material with density 4.70 g/cm^3. The space inside the shell is filled with a liquid having a density of 1.23 g/cm^3. (a) Find the mass m of the sphere, including its contents, as a function of a. (b) For what value of the variable a does m have its maximum possible value? (c) What is this maximum mass? (d) Explain whether the value from part (c) agrees with the result of a direct calculation of the mass of a solid sphere of uniform density made of the same material as the shell. (e) **What If?** Would the answer to part (a) change if the inner wall were not concentric with the outer wall?

35. Q|C Air is blown into a spherical balloon so that, when its radius is 6.50 cm, its radius is increasing at the rate 0.900 cm/s. (a) Find the rate at which the volume of the balloon is increasing. (b) If this volume flow rate of air entering the balloon is constant, at what rate will the radius be increasing when the radius is 13.0 cm? (c) Explain physically why the answer to part (b) is larger or smaller than 0.9 cm/s, if it is different.

36. In physics, it is important to use mathematical approximations. (a) Demonstrate that for small angles (< 20°)

$$\tan\alpha \approx \sin\alpha \approx \alpha = \frac{\pi\alpha'}{180°}$$

where α is in radians and α' is in degrees. (b) Use a calculator to find the largest angle for which tan α may be approximated by α with an error less than 10.0%.

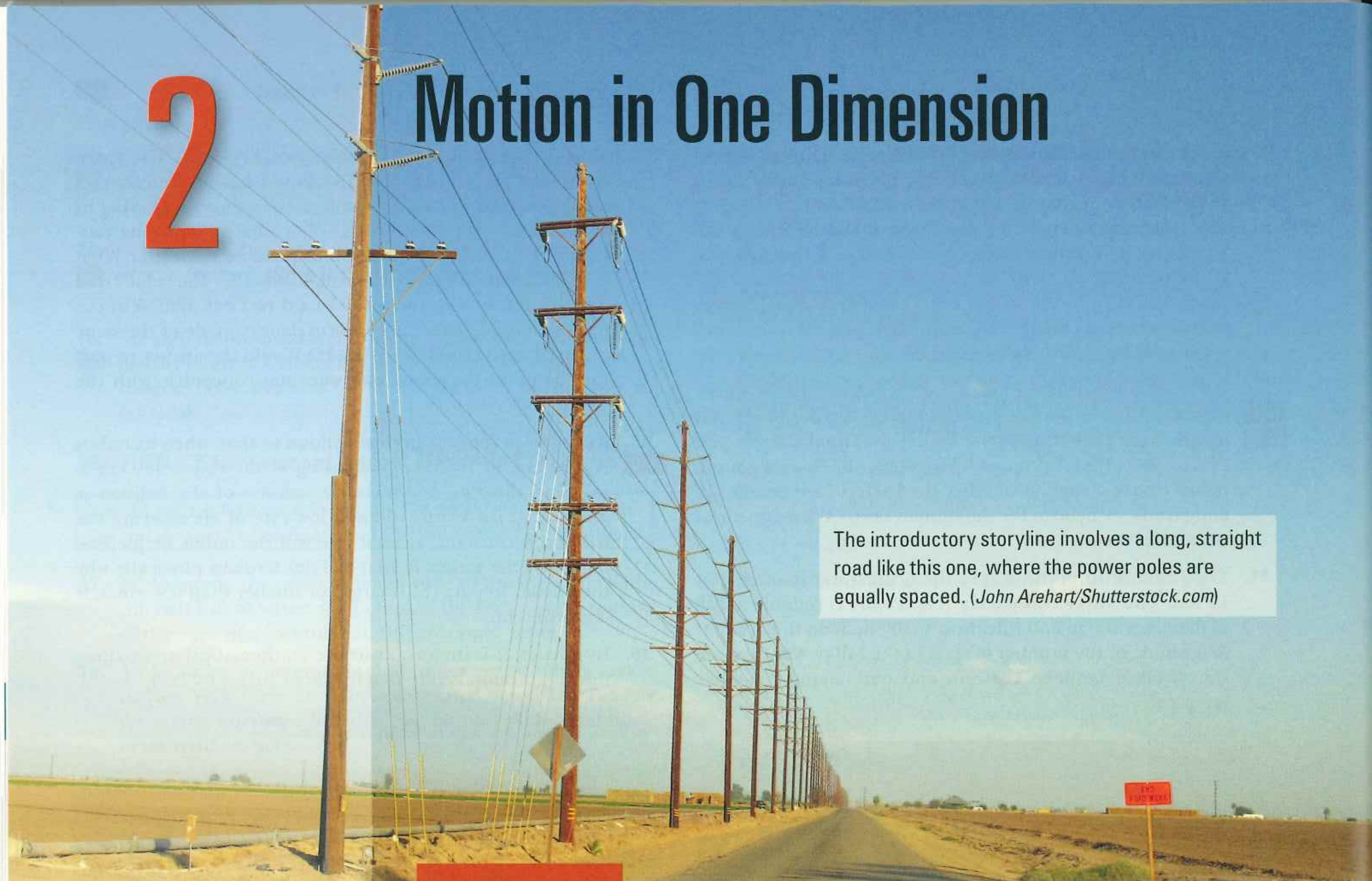
37. T The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.008\,00t^2$, where V is the volume of gas in millions of cubic feet and t is the time in months. Express this equation in units of cubic feet and seconds. Assume a month is 30.0 days.

38. GP A woman wishing to know the height of a mountain measures the angle of elevation of the mountaintop as 12.0°. After walking 1.00 km closer to the mountain on level ground, she finds the angle to be 14.0°. (a) Draw a picture of the problem, neglecting the height of the woman's eyes above the ground. *Hint:* Use two triangles. (b) Using the symbol y to represent the mountain height and the symbol x to represent the woman's original distance from the mountain, label the picture. (c) Using the labeled picture, write two trigonometric equations relating the two selected variables. (d) Find the height y.

CHALLENGE PROBLEM

39. S A woman stands at a horizontal distance x from a mountain and measures the angle of elevation of the mountaintop above the horizontal as θ. After walking a distance d closer to the mountain on level ground, she finds the angle to be ϕ. Find a general equation for the height y of the mountain in terms of d, ϕ, and θ, neglecting the height of her eyes above the ground.

2 Motion in One Dimension



The introductory storyline involves a long, straight road like this one, where the power poles are equally spaced. (*John Arehart/Shutterstock.com*)

STORYLINE **You are a passenger in a car being driven by a friend** down a straight road. You notice that the telephone poles, streetlight poles, or electric power poles on the side of the road are located at equal distances from each other. You pull out your smartphone and use it as a stopwatch to measure the time intervals required for you to pass between adjacent pairs of poles.[1] When your friend tells you that the car is moving at a fixed speed, you notice that all of these time intervals are the same. Now, the driver begins to slow down for a traffic light. You again measure the time intervals and find that each one is longer than the one before. After the car pulls away from the traffic light and speeds up, the time intervals between poles become shorter. Does this behavior make sense? When the car is moving at a constant speed again, you use the time interval between poles and the driving speed reported by your friend to calculate the distance between the poles. You excitedly tell your friend to pull over so you can pace out the distance between the poles. How accurate was your calculation?

CONNECTIONS We begin our study of physics with the topic of *kinematics*. In this broad topic, we generally investigate *motion*: the motion of objects without regard for interactions with the environment that influence the motion. Motion is what many of the early scientists studied. Early astronomers in Greece, China, the Middle East, and Central America observed the motion of objects in the night sky. Galileo Galilei studied the motion of objects rolling down inclined planes. Isaac Newton pondered the nature of falling objects. From everyday experience, we recognize that motion of an object represents a continuous change in the object's

[1]A number of specialized smartphone apps can be downloaded and used to make numerical measurements, such as speed and acceleration. In our storylines, however, we will restrict our smartphone use mostly to apps that are standard on the phone as purchased.

position. In this chapter, we will analyze the motion of an object along a straight line, like the car in the storyline. We will use measurements of length and time as described in Chapter 1 to quantify the motion. An object moving vertically and subject to gravity is an important application of one-dimensional motion, and will also be studied in this chapter. Remember our discussion of making models for physical situations in Section 1.2. In our study, we use the simplification model mentioned in that section and called the particle model, and describe the moving object as a particle regardless of its size. In general, a particle is a point-like object, that is, an object that has mass but is of infinitesimal size. In Section 1.2, we discussed the fact that the motion of the Earth around the Sun can be treated as if the Earth were a particle. We will return to this model for the Earth when we study planetary orbits in Chapter 13. As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles, without regard for the internal structure of the molecules; we will see this analysis in Chapter 20. For now, let us apply the particle model to a wide variety of moving objects in this chapter. An understanding of motion will be essential throughout the rest of this book: the motion of planets in Chapter 13 on gravity, the motion of electrons in electric circuits in Chapter 26, the motion of light waves in Chapter 34 on optics, the motion of quantum particles tunneling through barriers in Chapter 40.

2.1 Position, Velocity, and Speed of a Particle

◀ Position

A particle's **position** x is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system. The motion of a particle is completely known if the particle's position in space is known at all times.

Consider a car moving back and forth along the x axis as in Figure 2.1a (page 22). The numbers under the horizontal line are position markers for the car, similar to the equally spaced poles in the introductory storyline. When we begin collecting position data, the car is 30 m to the right of the reference position $x = 0$. We will use the particle model by identifying some point on the car, perhaps the front door handle, as a particle representing the entire car.

We start our clock, and once every 10 s we note the car's position. As you can see from Table 2.1, the car moves to the right (which we have defined as the positive direction) during the first 10 s of motion, from position Ⓐ to position Ⓑ. After Ⓑ, the position values begin to decrease, suggesting the car is backing up from position Ⓑ through position Ⓕ. In fact, at Ⓓ, 30 s after we start measuring, the car is at the origin of coordinates (see Fig. 2.1a). It continues moving to the left and is more than 50 m to the left of $x = 0$ when we stop recording information after our sixth data point. A graphical representation of this information is presented in Figure 2.1b. Such a plot is called a *position–time graph*.

TABLE 2.1 Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Notice the alternative representations of information, as discussed in Section 1.2, that we have used for the motion of the car. Figure 2.1a is a pictorial representation, whereas Figure 2.1b is a graphical representation. Table 2.1 is a tabular representation of the same information. The ultimate goal, as mentioned in Section 1.2, is a mathematical representation, which can be analyzed to solve for some requested piece of information.

In the introductory storyline, you observed the change in the position of your car relative to the power poles. The **displacement** Δx of a particle is defined as its change in position in some time interval. As the particle moves from an initial position x_i to a final position x_f, its displacement is given by

$$\Delta x \equiv x_f - x_i \qquad (2.1)$$

◀ Displacement

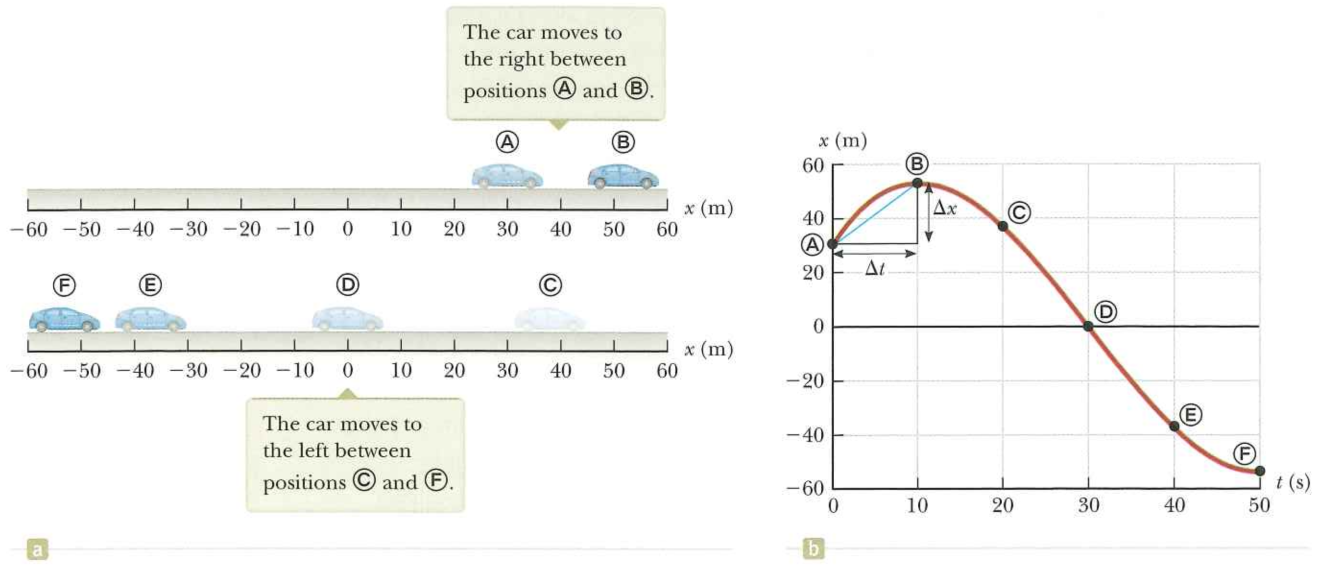


Figure 2.1 A car moves back and forth along a straight line. Because we are interested only in the car's translational motion, we can model it as a particle. Several representations of the information about the motion of the car can be used. Table 2.1 is a tabular representation of the information. (a) A pictorial representation of the motion of the car. (b) A graphical representation (position–time graph) of the motion of the car.

Figure 2.2 On this basketball court, players run back and forth for the entire game. The distance that the players run over the duration of the game is nonzero. The displacement of the players over the duration of the game is approximately zero because they keep returning to the same point over and over again.

We use the capital Greek letter delta (Δ) to denote the *change* in a quantity. From this definition, we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i. Given the data in Table 2.1, we can easily determine the displacement of the car for various time intervals.

It is very important to recognize the difference between displacement and distance traveled. **Distance** is the length of a path followed by a particle. Consider, for example, the basketball players in Figure 2.2. If a player runs from his own team's basket down the court to the other team's basket and then returns to his own basket, the *displacement* of the player during this time interval is zero because he ended up at the same point as he started: $x_f = x_i$, so $\Delta x = 0$. During this time interval, however, he moved through a *distance* of twice the length of the basketball court. Distance is always represented as a positive number, whereas displacement can be either positive or negative.

Displacement is an example of a vector quantity. Many other physical quantities, including position, velocity, and acceleration, also are vectors. In general, a **vector quantity** requires the specification of both direction and magnitude. For example, in the case of the car in Figure 2.1, by how much did the position of the car change (*magnitude*) and in what *direction*—forward or backward? By contrast, a **scalar quantity** has a numerical value and no direction. Distance is a scalar: how far did the car move, as measured by its odometer, in a certain time interval? In this chapter, we use positive (+) and negative (−) signs to indicate vector direction. For example, for horizontal motion let us arbitrarily specify to the right as being the positive direction. It follows that any object always moving to the right undergoes a positive displacement $\Delta x > 0$, and any object moving to the left undergoes a negative displacement so that $\Delta x < 0$. We shall treat vector quantities in greater detail in Chapter 3.

One very important point has not yet been mentioned. Notice that the data in Table 2.1 result only in the six data points in the graph in Figure 2.1b. Therefore, the motion of the particle is not completely known because we don't know its position at *all* times. The smooth curve drawn through the six points in the graph is only a *possibility* of the actual motion of the car. We only have information about six instants of time; we have no idea what happened between the data points. The smooth curve is a *guess* as to what happened, but keep in mind that it is *only* a guess. If the smooth curve does represent the actual motion of the car, the graph contains complete information about the entire 50-s interval during which we watch the car move.

QUICK QUIZ 2.1 Which of the following choices best describes what can be determined exactly from Table 2.1 and Figure 2.1 for the entire 50-s interval? (a) The distance the car moved. (b) The displacement of the car. (c) Both (a) and (b). (d) Neither (a) nor (b).

It is much easier to see changes in position from the graph than from a verbal description or even a table of numbers. For example, it is clear that the car covers more ground during the middle of the 50-s interval than at the end. Between positions Ⓒ and Ⓓ, the car changes position by almost 40 m, but during the last 10 s, between positions Ⓔ and Ⓕ, it changes position by less than half that much. A common way of comparing these different motions is to divide the displacement Δx that occurs between two clock readings by the value of that particular time interval Δt. The result turns out to be a very useful ratio, one that we shall use many times. This ratio has been given a special name: the *average velocity*. The **average velocity** $v_{x,\text{avg}}$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$v_{x,\text{avg}} \equiv \frac{\Delta x}{\Delta t} \qquad (2.2)$$

◀ Average velocity

where the subscript x indicates motion along the x axis. From this definition we see that average velocity has dimensions of length divided by time (L/T), or meters per second in SI units.

The average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.) If the coordinate of the particle increases in time (that is, if $x_f > x_i$), Δx is positive and $v_{x,\text{avg}} = \Delta x/\Delta t$ is positive. This case corresponds to a particle moving in the positive x direction, that is, toward larger values of x. If the coordinate decreases in time (that is, if $x_f < x_i$), Δx is negative and hence $v_{x,\text{avg}}$ is negative. This case corresponds to a particle moving in the negative x direction.

We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph in Figure 2.1b. This line forms the hypotenuse of a right triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$, which is what we have defined as average velocity in Equation 2.2. For example, the line between positions Ⓐ and Ⓑ in Figure 2.1b has a slope equal to the average velocity of the car between those two times, $(52\text{ m} - 30\text{ m})/(10\text{ s} - 0) = 2.2\text{ m/s}$.

In everyday usage, the terms *speed* and *velocity* are interchangeable. In physics, however, there is a clear distinction between these two quantities. Consider a marathon runner who runs a distance d of more than 40 km and yet ends up at her starting point. Her total displacement is zero, so her average velocity is zero! Nonetheless, we need to be able to quantify how fast she was running. A slightly different ratio accomplishes that for us. The **average speed** v_{avg} of a particle, a scalar quantity, is defined as the total distance d traveled divided by the total time interval required to travel that distance:

$$v_{\text{avg}} \equiv \frac{d}{\Delta t} \qquad (2.3)$$

◀ Average speed

The SI unit of average speed is the same as the unit of average velocity: meters per second. Unlike average velocity, however, average speed has no direction and

PITFALL PREVENTION 2.1

Average Speed and Average Velocity The magnitude of the average velocity is *not* the average speed. For example, consider the marathon runner discussed before Equation 2.3. The magnitude of her average velocity is zero, but her average speed is clearly not zero.

is always expressed as a positive number. Notice the clear distinction between the definitions of average velocity and average speed: average velocity (Eq. 2.2) is the *displacement* divided by the time interval, whereas average speed (Eq. 2.3) is the *distance* divided by the time interval.

Knowledge of the average velocity or average speed of a particle does not provide information about the details of the trip. For example, suppose it takes you 45.0 s to travel 100 m down a long, straight hallway toward your departure gate at an airport. At the 100-m mark, you realize you missed the restroom, and you return back 25.0 m along the same hallway, taking 10.0 s to make the return trip. The magnitude of your average *velocity* is +75.0 m/55.0 s = +1.36 m/s. The average *speed* for your trip is 125 m/55.0 s = 2.27 m/s. You may have traveled at various speeds during the walk and, of course, you changed direction. Neither average velocity nor average speed provides information about these details.

QUICK QUIZ 2.2 Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval? **(a)** A particle moves in the $+x$ direction without reversing. **(b)** A particle moves in the $-x$ direction without reversing. **(c)** A particle moves in the $+x$ direction and then reverses the direction of its motion. **(d)** There are no conditions for which this is true.

Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

SOLUTION

Consult Figure 2.1 to form a mental image of the car and its motion. We model the car as a particle. From the position–time graph given in Figure 2.1b, notice that $x_Ⓐ = 30$ m at $t_Ⓐ = 0$ s and that $x_Ⓕ = -53$ m at $t_Ⓕ = 50$ s.

Use Equation 2.1 to find the displacement of the car:

$$\Delta x = x_Ⓕ - x_Ⓐ = -53 \text{ m} - 30 \text{ m} = -83 \text{ m}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that it is the correct answer.

Use Equation 2.2 to find the car's average velocity:

$$v_{x,\text{avg}} = \frac{x_Ⓕ - x_Ⓐ}{t_Ⓕ - t_Ⓐ}$$

$$= \frac{-53 \text{ m} - 30 \text{ m}}{50 \text{ s} - 0 \text{ s}} = \frac{-83 \text{ m}}{50 \text{ s}} = -1.7 \text{ m/s}$$

We cannot unambiguously find the average speed of the car from the data in Table 2.1 because we do not have information about the positions of the car between the data points. If we adopt the assumption that the details of the car's position are described by the curve in Figure 2.1b, the distance traveled is 22 m (from Ⓐ to Ⓑ) plus 105 m (from Ⓑ to Ⓕ), for a total of 127 m.

Use Equation 2.3 to find the car's average speed:

$$v_{\text{avg}} = \frac{127 \text{ m}}{50 \text{ s}} = 2.5 \text{ m/s}$$

Notice that the average speed is positive, as it must be. Suppose the red-brown curve in Figure 2.1b were different so that between 0 s and 10 s it went from Ⓐ up to 100 m and then came back down to Ⓑ. The average speed of the car would change because the distance is different, but the average velocity would not change.

2.2 Instantaneous Velocity and Speed

Often we need to know the velocity of a particle at a particular instant in time t rather than the average velocity over a finite time interval Δt. In other words, you would like to be able to specify your velocity just as precisely as you can specify your position by noting what is happening at a specific clock reading, that is, at some

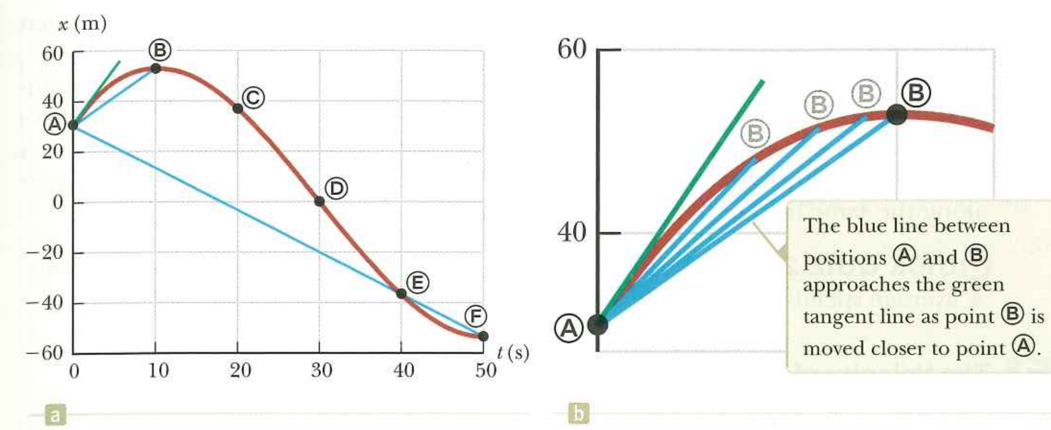


Figure 2.3 (a) Graph representing the motion of the car in Figure 2.1. (b) An enlargement of the upper-left-hand corner of the graph.

specific instant. What does it mean to talk about how quickly something is moving if we "freeze time" and talk only about an individual instant? If the time interval has a value of zero, the displacement of the object is also zero, so the average velocity from Equation 2.2 would seem to be 0/0. How do we evaluate that ratio? In the late 1600s, with the invention of calculus, scientists began to understand how to answer that question and describe an object's motion at any moment in time.

To see how that is done, consider Figure 2.3a, which is a reproduction of the graph in Figure 2.1b. What is the particle's velocity at $t = 0$? We have already discussed the average velocity for the interval during which the car moved from position Ⓐ to position Ⓑ (given by the slope of the blue line) and for the interval during which it moved from Ⓐ to Ⓕ (represented by the slope of the longer blue line and calculated in Example 2.1). The car starts out by moving to the right, which we defined to be the positive direction. Therefore, being positive, the value of the average velocity during the interval from Ⓐ to Ⓑ is more representative of the initial velocity than is the value of the average velocity during the interval from Ⓐ to Ⓕ, which we determined to be negative in Example 2.1. Now let us focus on the short blue line and imagine sliding point Ⓑ to the left along the curve, toward point Ⓐ, as in Figure 2.3b. The line between the points becomes steeper and steeper, and as the two points become extremely close together, the line becomes a tangent line to the curve, indicated by the green line in Figure 2.3b. The slope of this tangent line represents the velocity of the car at point Ⓐ. What we have done is determine the *instantaneous velocity* at that moment. In other words, the **instantaneous velocity** v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:[2]

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad (2.4)$$

In calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.5)$$

◄ Instantaneous velocity

The instantaneous velocity can be positive, negative, or zero. When the slope of the position–time graph is positive, such as at any time during the first 10 s in Figure 2.3, v_x is positive and the car is moving toward larger values of x. After point Ⓑ, v_x is negative because the slope is negative and the car is moving toward smaller values of x. At point Ⓑ, the slope and the instantaneous velocity are zero and the car is momentarily at rest.

PITFALL PREVENTION 2.2

Slopes of Graphs In any graph of physical data, the *slope* represents the ratio of the change in the quantity represented on the vertical axis to the change in the quantity represented on the horizontal axis. Remember that *a slope has units* (unless both axes have the same units). The units of slope in Figures 2.1b and 2.3 are meters per second, the units of velocity.

PITFALL PREVENTION 2.3

Instantaneous Speed and Instantaneous Velocity In Pitfall Prevention 2.1, we argued that the magnitude of the average velocity is not the average speed. The magnitude of the instantaneous velocity, however, *is* the instantaneous speed. In an infinitesimal time interval, the magnitude of the displacement is equal to the distance traveled by the particle.

[2]As mentioned previously, the displacement Δx also approaches zero as Δt approaches zero, so the ratio $\Delta x/\Delta t$ looks like 0/0. The ratio can be evaluated in the limit in this situation, however. As Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the slope of the line tangent to the x-versus-t curve.

From here on, we use the word *velocity* to designate instantaneous velocity. When we are interested in *average velocity,* we shall always use the adjective *average.*

The **instantaneous speed** of a particle is defined as the magnitude of its instantaneous velocity. As with average speed, instantaneous speed has no direction associated with it. For example, if one particle has an instantaneous velocity of +25 m/s along a given line and another particle has an instantaneous velocity of −25 m/s along the same line, both have a speed[3] of 25 m/s.

QUICK QUIZ 2.3 Are officers in the highway patrol more interested in **(a)** your average speed or **(b)** your instantaneous speed as you drive?

Conceptual Example 2.2 The Velocity of Different Objects

Consider the following one-dimensional motions: **(A)** a ball thrown directly upward rises to a highest point and falls back into the thrower's hand; **(B)** a race car starts from rest and speeds up to 100 m/s; and **(C)** a spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity has the same value as the average velocity over the entire motion? If so, identify the point(s).

SOLUTION

(A) The average velocity for the thrown ball is zero because the ball returns to the starting point; therefore, its displacement is zero. There is one point at which the instantaneous velocity is zero: at the top of the motion.

(B) The car's average velocity cannot be evaluated unambiguously with the information given, but it must have some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity over the entire motion.

(C) Because the spacecraft's instantaneous velocity is constant, its instantaneous velocity at *any* time and its average velocity over *any* time interval are the same.

Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[4] The position–time graph for this motion is shown in Figure 2.4a. Because the position of the particle is given by a mathematical function, the motion of the particle is known at all times, *unlike* that of the car in Figure 2.1, where data is only provided at six instants of time. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

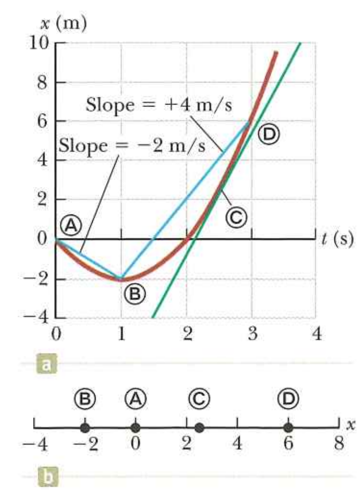


Figure 2.4 (Example 2.3) (a) Position–time graph for a particle having an x coordinate that varies in time according to the expression $x = -4t + 2t^2$. (b) The particle moves in one dimension along the x axis.

SOLUTION

From the graph in Figure 2.4a, form a mental representation of the particle's motion. Keep in mind that the particle does not move in a curved path in space such as that shown by the red-brown curve in the graphical representation. The particle moves only along the x axis in one dimension as shown in Figure 2.4b. At $t = 0$, is it moving to the right or to the left?

During the first time interval, the slope is negative and hence the average velocity is negative. Therefore, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

continued

[3] As with velocity, we drop the adjective for instantaneous speed. *Speed* means "instantaneous speed."

[4] Simply to make it easier to read, we write the expression as $x = -4t + 2t^2$ rather than as $x = (-4.00 \text{ m/s})t + (2.00 \text{ m/s}^2)t^{2.00}$. When an equation summarizes measurements, consider its coefficients and exponents to have as many significant figures as other data quoted in a problem. Consider its coefficients to have the units required for dimensional consistency. When we start our clocks at $t = 0$, we usually do not mean to limit the precision to a single digit. Consider any zero value in this book to have as many significant figures as you need.

2.3 continued

In the first time interval, set $t_i = t_{\text{Ⓐ}} = 0$ and $t_f = t_{\text{Ⓑ}} = 1$ s. Substitute these values into $x = -4t + 2t^2$ and use Equation 2.1 to find the displacement:

$$\Delta x_{\text{Ⓐ}\to\text{Ⓑ}} = x_f - x_i = x_{\text{Ⓑ}} - x_{\text{Ⓐ}}$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = -2 \text{ m}$$

For the second time interval ($t = 1$ s to $t = 3$ s), set $t_i = t_{\text{Ⓑ}} = 1$ s and $t_f = t_{\text{Ⓓ}} = 3$ s:

$$\Delta x_{\text{Ⓑ}\to\text{Ⓓ}} = x_f - x_i = x_{\text{Ⓓ}} - x_{\text{Ⓑ}}$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = +8 \text{ m}$$

These displacements can also be read directly from the position–time graph.

(B) Calculate the average velocity during these two time intervals.

SOLUTION

In the first time interval, use Equation 2.2 with $\Delta t = t_f - t_i = t_{\text{Ⓑ}} - t_{\text{Ⓐ}} = 1$ s:

$$v_{x,\text{avg}\,(\text{Ⓐ}\to\text{Ⓑ})} = \frac{\Delta x_{\text{Ⓐ}\to\text{Ⓑ}}}{\Delta t} = \frac{-2 \text{ m}}{1 \text{ s}} = -2 \text{ m/s}$$

In the second time interval, $\Delta t = 2$ s:

$$v_{x,\text{avg}\,(\text{Ⓑ}\to\text{Ⓓ})} = \frac{\Delta x_{\text{Ⓑ}\to\text{Ⓓ}}}{\Delta t} = \frac{8 \text{ m}}{2 \text{ s}} = +4 \text{ m/s}$$

These values are the same as the slopes of the blue lines joining these points in Figure 2.4a.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

SOLUTION

Calculate the slope of the green line at $t = 2.5$ s (point Ⓒ) in Figure 2.4a by reading position and time values for the ends of the green line from the graph:

$$v_x = \frac{10 \text{ m} - (-4 \text{ m})}{3.8 \text{ s} - 1.5 \text{ s}} = +6 \text{ m/s}$$

Notice that this instantaneous velocity is on the same order of magnitude as our previous results, that is, a few meters per second. Is that what you would have expected?

2.3 Analysis Model: Particle Under Constant Velocity

In Section 1.2 we discussed the importance of making models. As mentioned there, a particularly important model used in the solution to physics problems is an *analysis model.* **An analysis model is a common situation that occurs time and again when solving physics problems.** Because it represents a common situation, it also represents a common type of problem that we have solved before. When you identify an analysis model in a new problem, the solution to the new problem can be modeled after that of the previously solved problem. Analysis models help us to recognize those common situations and guide us toward a solution to the problem. The form that an analysis model takes is a description of either (1) the behavior of some physical entity or (2) the interaction between that entity and the environment. When you encounter a new problem, you should identify the fundamental details of the problem, ignore details that are not important, and attempt to recognize which of the situations you have already seen that might be used as a model for the new problem. For example, suppose an automobile is moving along a straight freeway at a constant speed. Is it important that it is an automobile? Is it important that it is a freeway? If the answers to both questions are no, but the car moves in a straight line at constant speed, we model the automobile as a *particle under constant velocity,* which we will discuss in this section. Once the problem has been modeled, it is no longer about an automobile. It is about a particle undergoing a certain type of motion, a motion that we have studied before.

◀ Analysis model

This method is somewhat similar to the common practice in the legal profession of finding "legal precedents." If a previously resolved case can be found that is very similar legally to the current one, it is used as a model and an argument is made in court to link them logically. The finding in the previous case can then be used to sway the finding in the current case. We will do something similar in physics. For a given problem, we search for a "physics precedent," a model with which we are already familiar and that can be applied to the current problem.

All of the analysis models that we will develop are based on four fundamental simplification models. The first of the four is the particle model discussed in the introduction to this chapter. We will look at a particle under various behaviors and environmental interactions. Further analysis models are introduced in later chapters based on simplification models of a *system*, a *rigid object*, and a *wave*. Once we have introduced these analysis models, we shall see that they appear again and again in different problem situations.

When solving a problem, you should avoid browsing through the chapter looking for an equation that contains the unknown variable that is requested in the problem. In many cases, the equation you find may have nothing to do with the problem you are attempting to solve. It is *much* better to take this first step: **Identify the analysis model that is appropriate for the problem.** To do so, think carefully about what is going on in the problem and match it to a situation you have seen before. Once the analysis model is identified, there are a small number of equations from which to choose that are appropriate for that model, sometimes only one equation. Therefore, **the model tells you which equation(s) to use for the mathematical representation.**

Let us use Equation 2.2 to build our first analysis model for solving problems. We imagine a particle moving with a constant velocity. The model of a **particle under constant velocity** can be applied in *any* situation in which an entity that can be modeled as a particle is moving with constant velocity. This situation occurs frequently, so this model is important.

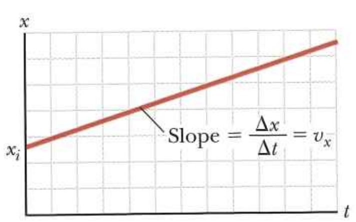


Figure 2.5 Position–time graph for a particle under constant velocity. The value of the constant velocity is the slope of the line.

If the velocity of a particle is constant, its instantaneous velocity at any instant during a time interval is the same as the average velocity over the interval. That is, $v_x = v_{x,\text{avg}}$. Therefore, substituting v_x for $v_{x,\text{avg}}$ in Equation 2.2 gives us an equation to be used in the mathematical representation of this situation:

$$v_x = \frac{\Delta x}{\Delta t} \tag{2.6}$$

Remembering that $\Delta x = x_f - x_i$, we see that $v_x = (x_f - x_i)/\Delta t$, or

$$x_f = x_i + v_x \Delta t$$

This equation tells us that the position of the particle is given by the sum of its original position x_i at time $t = 0$ plus the displacement $v_x \Delta t$ that occurs during the time interval Δt. In practice, we usually choose the time at the beginning of the interval to be $t_i = 0$ and the time at the end of the interval to be $t_f = t$, so our equation becomes

Position as a function of time for the particle under constant velocity model

$$x_f = x_i + v_x t \quad \text{(for constant } v_x\text{)} \tag{2.7}$$

Equations 2.6 and 2.7 are the primary equations used in the model of a particle under constant velocity. Whenever you have identified the analysis model in a problem to be the particle under constant velocity, you can immediately turn to these equations.

Figure 2.5 is a graphical representation of the particle under constant velocity. On this position–time graph, the slope of the line representing the motion is constant and equal to the magnitude of the velocity. Equation 2.7, which is the equation of a straight line, is the mathematical representation of the particle under constant velocity model. The slope of the straight line is v_x and the y intercept is x_i in both representations.

In the opening storyline, the particle under constant velocity model was represented by the part of the motion taking place at "fixed speed." You found in the storyline that the time intervals between poles were always the same in this case. Is this result consistent with Equation 2.7? Example 2.4 below shows a numerical application of the particle under constant velocity model.

Example 2.4 Modeling a Runner as a Particle

A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics*.) She determines the velocity of an experimental subject while he runs along a straight line at a constant rate. The kinesiologist starts the stopwatch at the moment the runner passes a given point and stops it after the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4.0 s.

(A) What is the runner's velocity?

SOLUTION

We model the moving runner as a particle because the size of the runner and the movement of arms and legs are unnecessary details. Because the problem states that the subject runs "at a constant rate," we can model him as a *particle under constant velocity*.

Having identified the model, we can use Equation 2.6 to find the constant velocity of the runner:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{20\text{ m} - 0}{4.0\text{ s}} = 5.0\text{ m/s}$$

(B) If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s have passed?

SOLUTION

Use Equation 2.7 and the velocity found in part (A) to find the position of the particle at time $t = 10$ s:

$$x_f = x_i + v_x t = 0 + (5.0\text{ m/s})(10\text{ s}) = 50\text{ m}$$

Is the result for part (A) a reasonable speed for a human? How does it compare to world-record speeds in 100-m and 200-m sprints? Notice the value in part (B) is more than twice that of the 20-m position at which the stopwatch was stopped. Is this value consistent with the time of 10 s being more than twice the time of 4.0 s?

The mathematical manipulations for the particle under constant velocity stem from Equation 2.6 and its descendent, Equation 2.7. These equations can be used to solve for any variable in the equations that happens to be unknown if the other variables are known. For example, in part (B) of Example 2.4, we find the position when the velocity and the time are known. Similarly, if we know the velocity and the final position, we could use Equation 2.7 to find the time at which the runner is at this position.

A particle under constant velocity moves with a constant speed along a straight line. Now consider a particle moving with a constant speed through a distance d along a *curved* path. As we will see in Section 2.5 below, a change in the direction of motion of a particle signifies a change in the velocity of a particle even though its speed is constant; there is a change in the speed *vector.* Therefore, our particle moving along a curved path is not represented by the particle under constant velocity model. However, it can be represented with the model of a **particle under constant speed.** The primary equation for this model is Equation 2.3, with the average speed v_{avg} replaced by the constant speed v:

$$v = \frac{d}{\Delta t} \tag{2.8}$$

As an example, imagine a particle moving at a constant speed in a circular path. If the speed is 5.00 m/s and the radius of the path is 10.0 m, we can calculate the time interval required to complete one trip around the circle:

$$v = \frac{d}{\Delta t} \rightarrow \Delta t = \frac{d}{v} = \frac{2\pi r}{v} = \frac{2\pi(10.0\text{ m})}{5.00\text{ m/s}} = 12.6\text{ s}$$

ANALYSIS MODEL **Particle Under Constant Velocity**

Imagine a moving object that can be modeled as a particle. If it moves at a constant speed through a displacement Δx in a straight line in a time interval Δt, its constant velocity is

$$v_x = \frac{\Delta x}{\Delta t} \quad (2.6)$$

The position of the particle as a function of time is given by

$$x_f = x_i + v_x t \quad (2.7)$$

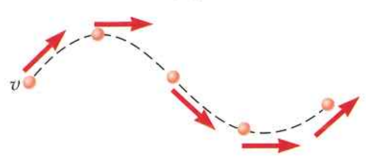
Examples:

- a meteoroid traveling through gravity-free space
- a car traveling at a constant speed on a straight highway
- a runner traveling at constant speed on a perfectly straight path
- an object moving at terminal speed through a viscous medium (Chapter 6)

ANALYSIS MODEL **Particle Under Constant Speed**

Imagine a moving object that can be modeled as a particle. If it moves at a constant speed through a distance d along a straight line or a curved path in a time interval Δt, its constant speed is

$$v = \frac{d}{\Delta t} \quad (2.8)$$



Examples:

- a planet traveling around a perfectly circular orbit
- a car traveling at a constant speed on a curved racetrack
- a runner traveling at constant speed on a curved path
- a charged particle moving through a uniform magnetic field (Chapter 28)

2.4 The Analysis Model Approach to Problem Solving

We have just seen our first analysis models: the particle under constant velocity and the particle under constant speed. Now, what do we do with these models? The analysis models fit into a general method of solving problems that we describe below. In particular, pay attention to the "Categorize" step in the discussion below. That is where you identify the analysis model to be applied to the problem. After that, the problem is solved using the equation or equations that you have already learned to be associated with that model. This is the way physicists approach complex situations and complicated problems, and break them into manageable pieces. It is an extremely useful skill for you to learn. It may look complicated at first, but it will become easier and of second nature as you practice it!

Conceptualize

- The first things to do when approaching a problem are to *think about* and *understand* the situation. Study carefully any representations of the information (for example, diagrams, graphs, tables, or photographs) that accompany the problem. Imagine a movie, running in your mind, of what happens in the problem: the mental representation.
- If a pictorial representation is not provided, you should almost always make a quick drawing of the situation. Indicate any known values, perhaps in a table or directly on your sketch.
- Now focus on what algebraic or numerical information is given in the problem. Carefully read the problem statement, looking for key phrases such as "starts from rest" ($v_i = 0$) or "stops" ($v_f = 0$).

- Now focus on the expected result of solving the problem. Exactly what is the question asking? Will the final result be numerical, algebraic, or verbal? Do you know what units to expect?
- Don't forget to incorporate information from your own experiences and common sense. What should a reasonable answer look like? For example, you wouldn't expect to calculate the speed of an automobile to be 5×10^6 m/s.

Categorize

- Once you have a good idea of what the problem is about, you need to *simplify* the problem. Use a simplification model to remove the details that are not important to the solution. For example, model a moving object as a particle. If appropriate, ignore air resistance or friction between a sliding object and a surface.
- Once the problem is simplified, it is important to *categorize* the problem in one of two ways. Is it a simple *substitution problem* such that numbers can be substituted into a simple equation or a definition? If so, the problem is likely to be finished when this substitution is done. If not, you face what we call an *analysis problem:* the situation must be analyzed more deeply to generate an appropriate equation and reach a solution.
- If it is an analysis problem, it needs to be categorized further. Have you seen this type of problem before? Does it fall into the growing list of types of problems that you have solved previously? If so, identify any *analysis model(s)* appropriate for the problem to prepare for the Analyze step below. Being able to classify a problem with an analysis model can make it much easier to lay out a plan to solve it.

Analyze

- Now you must analyze the problem and strive for a mathematical solution. Because you have already categorized the problem and identified an analysis model, it should not be too difficult to select relevant equations that apply to the type of situation in the problem. For example, if the problem involves a particle under constant velocity, Equation 2.7 is relevant.
- Use algebra (and calculus, if necessary) to solve symbolically for the unknown variable in terms of what is given. Finally, substitute in the appropriate numbers, calculate the result, and round it to the proper number of significant figures.

Finalize

- Examine your numerical answer. Does it have the correct units? Does it meet your expectations from your conceptualization of the problem? What about the algebraic form of the result? Does it make sense? Examine the variables in the problem to see whether the answer would change in a physically meaningful way if the variables were drastically increased or decreased or even became zero. Looking at limiting cases to see whether they yield expected values is a very useful way to make sure that you are obtaining reasonable results.
- Think about how this problem compared with others you have solved. How was it similar? In what critical ways did it differ? Why was this problem assigned? Can you figure out what you have learned by doing it? If it is a new category of problem, be sure you understand it so that you can use it as a model for solving similar problems in the future.

When solving complex problems, you may need to identify a series of subproblems and apply the Analysis Model Approach to each. For simple problems, you probably don't need this approach. When you are trying to solve a problem and you don't know what to do next, however, remember the steps in the approach and use them as a guide.

In the rest of this book, we will label the *Conceptualize, Categorize, Analyze,* and *Finalize* steps in the worked examples. If a worked example is identified as a substitution problem in the *Categorize* step, there will generally not be *Analyze* and *Finalize* sections labeled in the solution.

To show how to apply this approach, we reproduce Example 2.4 below, with the steps of the approach labeled.

Example 2.4 Modeling a Runner as a Particle

A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics.*) She determines the velocity of an experimental subject while he runs along a straight line at a constant rate. The kinesiologist starts the stopwatch at the moment the runner passes a given point and stops it after the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4.0 s.

(A) What is the runner's velocity?

SOLUTION

Conceptualize We model the moving runner as a particle because the size of the runner and the movement of arms and legs are unnecessary details.

Categorize Because the problem states that the subject runs "at a constant rate," we can model him as a *particle under constant velocity.*

Analyze Having identified the model, we can use Equation 2.6 to find the constant velocity of the runner:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{20\text{ m} - 0}{4.0\text{ s}} = 5.0\text{ m/s}$$

(B) If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s have passed?

SOLUTION

Use Equation 2.7 and the velocity found in part (A) to find the position of the particle at time $t = 10$ s:

$$x_f = x_i + v_x t = 0 + (5.0\text{ m/s})(10\text{ s}) = 50\text{ m}$$

Finalize Is the result for part (A) a reasonable speed for a human? How does it compare to world-record speeds in 100-m and 200-m sprints? Notice the value in part (B) is more than twice that of the 20-m position at which the stopwatch was stopped. Is this value consistent with the time of 10 s being more than twice the time of 4.0 s?

2.5 Acceleration

In Example 2.3, we worked with a common situation in which the velocity of a particle changes while the particle is moving. When the velocity of a particle changes with time, the particle is said to be *accelerating.* For example, the magnitude of a car's velocity increases when you step on the gas and decreases when you apply the brakes. Both of these actions result in an acceleration of the car. Let us see how to quantify acceleration.

Suppose an object that can be modeled as a particle moving along the x axis has an initial velocity v_{xi} at time t_i at position Ⓐ and a final velocity v_{xf} at time t_f at position Ⓑ as in Figure 2.6a. The red-brown curve in Figure 2.6b shows how the velocity varies with time. The **average acceleration** $a_{x,\text{avg}}$ of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurs:

Average acceleration ▶

$$a_{x,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \tag{2.9}$$

As with velocity, when the motion being analyzed is one dimensional, we can use positive and negative signs to indicate the direction of the acceleration. Because the dimensions of velocity are L/T and the dimension of time is T, acceleration has

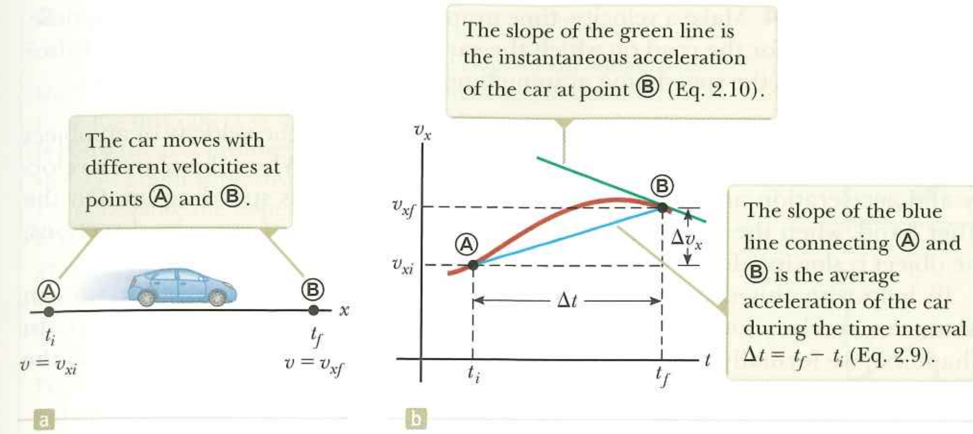


Figure 2.6 (a) A car, modeled as a particle, moving along the x axis from Ⓐ to Ⓑ, has velocity v_{xi} at $t = t_i$ and velocity v_{xf} at $t = t_f$. (b) Velocity–time graph (red-brown) for the particle moving in a straight line.

dimensions of length divided by time squared, or L/T². The SI unit of acceleration is meters per second squared (m/s²). It might be easier to interpret these units if you think of them as meters per second per second. For example, suppose an object has an acceleration of +2 m/s². You can interpret this value by forming a mental image of the object having a velocity that is along a straight line and is increasing by 2 m/s during every time interval of 1 s. If the object starts from rest, you should be able to picture it moving at a velocity of +2 m/s after 1 s, at +4 m/s after 2 s, and so on.

When your friend sped up from the traffic light in the opening storyline, you found that the time intervals between poles on the side of the road decreased. Is that result consistent with your expectations? Each new displacement between poles is undertaken at a higher speed, so the time intervals between poles become smaller.

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the **instantaneous acceleration** as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in Section 2.2. If we imagine that point Ⓐ is brought closer and closer to point Ⓑ in Figure 2.6a and we take the limit of $\Delta v_x/\Delta t$ as Δt approaches zero, we obtain the instantaneous acceleration at point Ⓑ:

◀ Instantaneous acceleration

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \tag{2.10}$$

That is, the instantaneous acceleration equals the derivative of the velocity with respect to time, which by definition is the slope of the velocity–time graph. The slope of the green line in Figure 2.6b is equal to the instantaneous acceleration at point Ⓑ. Notice that Figure 2.6b is a *velocity–time* graph, not a *position–time* graph like Figures 2.1b, 2.3, 2.4, and 2.5. Therefore, we see that just as the velocity of a moving particle is the slope at a point on the particle's x–t graph, the acceleration of a particle is the slope at a point on the particle's v_x–t graph. One can interpret the derivative of the velocity with respect to time as the time rate of change of velocity. If a_x is positive, the acceleration is in the positive x direction; if a_x is negative, the acceleration is in the negative x direction.

Figure 2.7 illustrates how an acceleration–time graph is related to a velocity–time graph. The acceleration at any time is the slope of the velocity–time graph at that time. Positive values of acceleration correspond to those points in Figure 2.7a where the velocity is increasing in the positive x direction. The acceleration reaches a maximum at time $t_Ⓐ$, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time $t_Ⓑ$, when the velocity is a maximum (that is, when the slope of the v_x–t graph is zero). The acceleration is negative when the velocity is decreasing in the positive x direction, and it reaches its most negative value at time $t_Ⓒ$.

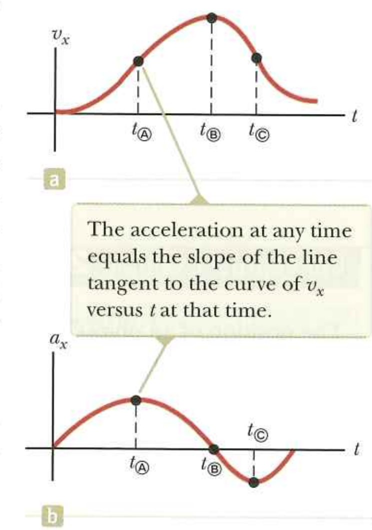


Figure 2.7 (a) The velocity–time graph for a particle moving along the x axis. (b) The instantaneous acceleration can be obtained from the velocity–time graph.

QUICK QUIZ 2.4 Make a velocity–time graph for the car in Figure 2.1a. Suppose the speed limit for the road on which the car is driving is 30 km/h. True or False? The car exceeds the speed limit at some time within the time interval 0–50 s.

For the case of motion in a straight line, the direction of the velocity of an object and the direction of its acceleration are related as follows. When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.

To help with this discussion of the signs of velocity and acceleration, we can relate the acceleration of an object to the total *force* exerted on the object. In Chapter 5, we formally establish that **the force on an object is proportional to the acceleration of the object:**

$$F_x \propto a_x \qquad (2.11)$$

This proportionality indicates that acceleration is caused by force. Furthermore, force and acceleration are both vectors, and the vectors are in the same direction. Therefore, let us think about the signs of velocity and acceleration by imagining a force applied to an object and causing it to accelerate. Let us assume the velocity and acceleration are in the same direction. This situation corresponds to an object that experiences a force acting in the same direction as its velocity. In this case, the object speeds up! Now suppose the velocity and acceleration are in opposite directions. In this situation, the object moves in some direction and experiences a force acting in the opposite direction. Therefore, the object slows down! It is very useful to equate the direction of the acceleration to the direction of a force because it is easier from our everyday experience to think about what effect a force will have on an object than to think only in terms of the direction of the acceleration.

PITFALL PREVENTION 2.4
Negative Acceleration Keep in mind that *negative acceleration does not necessarily mean that an object is slowing down.* If the acceleration is negative and the velocity is negative, the object is speeding up!

PITFALL PREVENTION 2.5
Deceleration The word *deceleration* has the common popular connotation of *slowing down.* We will not use this word in this book because it confuses the definition we have given for negative acceleration.

QUICK QUIZ 2.5 If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down? **(a)** eastward **(b)** westward **(c)** neither eastward nor westward

From now on, we shall use the term *acceleration* to mean instantaneous acceleration. When we mean average acceleration, we shall always use the adjective *average.* Because $v_x = dx/dt$, the acceleration can also be written as

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad (2.12)$$

That is, in one-dimensional motion, the acceleration of a particle equals the *second derivative* of the particle's position x with respect to time.

Conceptual Example 2.5 Graphical Relationships Between x, v_x, and a_x

The position of an object moving along the x axis varies with time as in Figure 2.8a. Graph the velocity versus time and the acceleration versus time for the object.

SOLUTION

The velocity at any instant is the slope of the tangent to the x–t graph at that instant. Between $t = 0$ and $t = t_{\text{Ⓐ}}$, the slope of the x–t graph increases uniformly, so the velocity increases linearly as shown in Figure 2.8b. Between $t_{\text{Ⓐ}}$ and $t_{\text{Ⓑ}}$, the slope of the x–t graph is constant, so the velocity remains constant. Between $t_{\text{Ⓑ}}$ and $t_{\text{Ⓓ}}$, the slope of the x–t graph decreases, so the value of the velocity in the v_x–t graph decreases. At $t_{\text{Ⓓ}}$, the slope of the x–t graph is zero, so the velocity is zero at that instant. Between $t_{\text{Ⓓ}}$ and $t_{\text{Ⓔ}}$, the slope of the x–t graph and therefore the velocity are negative and decrease uniformly in this interval. In the interval $t_{\text{Ⓔ}}$ to $t_{\text{Ⓕ}}$, the slope of the x–t graph is still negative, and at $t_{\text{Ⓕ}}$ it goes to zero. Finally, after $t_{\text{Ⓕ}}$, the slope of the x–t graph is zero, meaning that the object is at rest for $t > t_{\text{Ⓕ}}$.

continued

2.5 continued

The acceleration at any instant is the slope of the tangent to the v_x–t graph at that instant. The graph of acceleration versus time for this object is shown in Figure 2.8c. The acceleration is constant and positive between 0 and $t_{\text{Ⓐ}}$, where the slope of the v_x–t graph is positive. It is zero between $t_{\text{Ⓐ}}$ and $t_{\text{Ⓑ}}$ and for $t > t_{\text{Ⓕ}}$ because the slope of the v_x–t graph is zero at these times. It is negative between $t_{\text{Ⓑ}}$ and $t_{\text{Ⓔ}}$ because the slope of the v_x–t graph is negative during this interval. Between $t_{\text{Ⓔ}}$ and $t_{\text{Ⓕ}}$, the acceleration is positive like it is between 0 and $t_{\text{Ⓐ}}$, but higher in value because the slope of the v_x–t graph is steeper.

Notice that the sudden changes in acceleration shown in Figure 2.8c are unphysical. Such instantaneous changes cannot occur in reality.

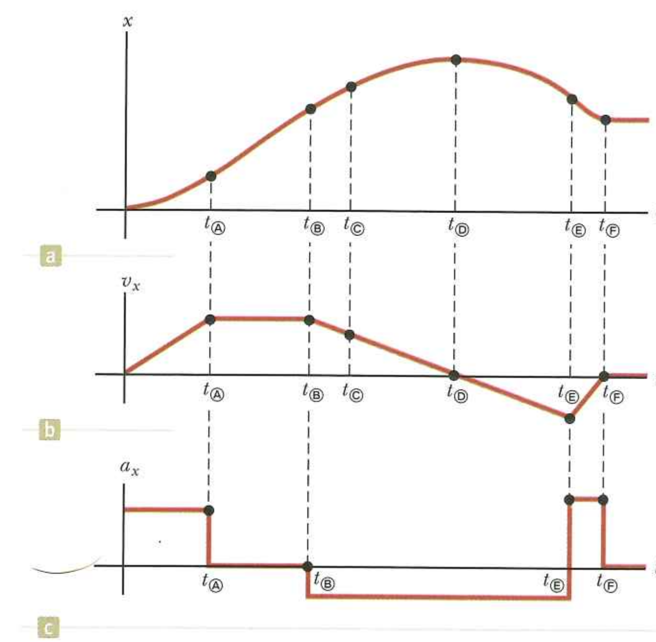


Figure 2.8 (Conceptual Example 2.5) (a) Position–time graph for an object moving along the x axis. (b) The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant. (c) The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

SOLUTION

Conceptualize Think about what the particle is doing from the mathematical representation. Is it moving at $t = 0$? In which direction? Does it speed up or slow down? Figure 2.9 is a v_x–t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x–t curve is negative, we expect the acceleration to be negative.

Categorize The solution to this problem does not require either of the analysis models we have developed so far, and can be solved with simple mathematics. Therefore, we categorize the problem as a substitution problem.

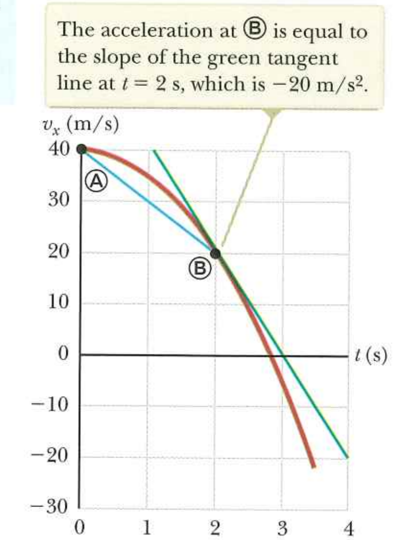


Figure 2.9 (Example 2.6) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = 40 - 5t^2$.

Find the velocities at $t_i = t_{\text{Ⓐ}} = 0$ and $t_f = t_{\text{Ⓑ}} = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{x\text{Ⓐ}} = 40 - 5t_{\text{Ⓐ}}^2 = 40 - 5(0)^2 = +40 \text{ m/s}$$

$$v_{x\text{Ⓑ}} = 40 - 5t_{\text{Ⓑ}}^2 = 40 - 5(2.0)^2 = +20 \text{ m/s}$$

Use Equation 2.9 to find the average acceleration in the specified time interval $\Delta t = t_{\text{Ⓑ}} - t_{\text{Ⓐ}} = 2.0$ s:

$$a_{x,\text{avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{x\text{Ⓑ}} - v_{x\text{Ⓐ}}}{t_{\text{Ⓑ}} - t_{\text{Ⓐ}}} = \frac{20 \text{ m/s} - 40 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$$

$$= -10 \text{ m/s}^2$$

The negative sign is consistent with our expectations: the average acceleration, represented by the slope of the blue line joining the initial and final points on the velocity–time graph, is negative.

continued

2.6 continued

(B) Determine the acceleration at $t = 2.0$ s.

SOLUTION

Knowing that the initial velocity at any time t is $v_{xi} = 40 - 5t^2$, find the velocity at any later time $t + \Delta t$:

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Find the change in velocity over the time interval Δt:

$$\Delta v_x = v_{xf} - v_{xi} = -10t\,\Delta t - 5(\Delta t)^2$$

To find the acceleration at any time t, divide this expression by Δt and take the limit of the result as Δt approaches zero:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\,\Delta t) = -10t$$

Substitute $t = 2.0$ s:

$$a_x = (-10)(2.0)\ \text{m/s}^2 = -20\ \text{m/s}^2$$

Because the velocity of the particle is positive and the acceleration is negative at this instant, the particle is slowing down.

Finalize Notice that the answers to parts (A) and (B) are different. The average acceleration in part (A) is the slope of the blue line in Figure 2.9 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in part (B) is the slope of the green line tangent to the curve at point Ⓑ. Notice also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.7.

So far, we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. If you are familiar with calculus, you should recognize that there are specific rules for taking derivatives. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly. For instance, one rule tells us that the derivative of any constant is zero. As another example, suppose x is proportional to some power of t such as in the expression

$$x = At^n$$

where A and n are constants. (This expression is a very common functional form.) The derivative of x with respect to t is

$$\frac{dx}{dt} = nAt^{n-1}$$

Applying these rules to Example 2.6, in which $v_x = 40 - 5t^2$, we quickly find that the acceleration is $a_x = dv_x/dt = -10t$, as we found in part (B) of the example.

2.6 Motion Diagrams

The concepts of velocity and acceleration are often confused with each other, but in fact they are quite different quantities. In forming a mental representation of a moving object, a pictorial representation called a *motion diagram* is sometimes useful to describe the velocity and acceleration while an object is in motion.

A motion diagram can be formed by imagining a *stroboscopic* photograph of a moving object, which shows several images of the object taken as the strobe light flashes at a constant rate. Figure 2.1a is a motion diagram for the car studied in Section 2.1. Figure 2.10 represents three sets of strobe photographs of cars moving along a straight roadway in a single direction, from left to right. The time intervals between flashes of the stroboscope are equal in each part of the diagram. So as to not confuse the two vector quantities, we use red arrows for velocity and purple arrows for acceleration in Figure 2.10. The arrows are shown at several instants during the motion of the object. Let us describe the motion of the car in each diagram.

In Figure 2.10a, the images of the car are equally spaced, showing us that the car moves through the same displacement in each time interval. This equal spacing is consistent with the car moving with *constant positive velocity* and *zero acceleration*. We could model the car as a particle and describe it with the particle under constant velocity model. The red velocity arrows are all of equal length, and there is no purple acceleration arrow shown because it is of length zero.

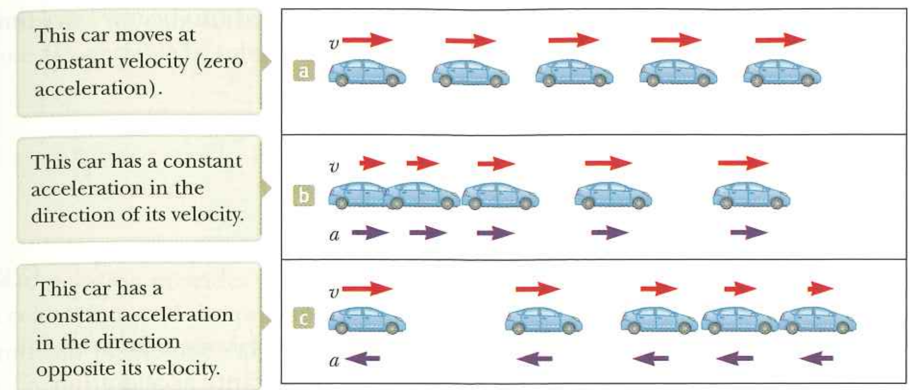


Figure 2.10 Motion diagrams of a car moving along a straight roadway in a single direction. The velocity at each instant is indicated by a red arrow, and the constant acceleration is indicated by a purple arrow.

In Figure 2.10b, the images become farther apart as time progresses. In this case, the red velocity arrows increase in length with time because the car's displacement between adjacent positions increases in time. These features suggest the car is moving with a *positive velocity* and a *positive acceleration*. The velocity and acceleration are in the same direction. In terms of our earlier force discussion, imagine a force pulling on the car in the same direction it is moving: it speeds up.

In Figure 2.10c, we can tell that the car slows as it moves to the right because its displacement between adjacent images decreases with time. This case suggests the car moves to the right with a negative acceleration. The lengths of the velocity arrows decrease in time and eventually reach zero. From this diagram, we see that the acceleration and velocity arrows are *not* in the same direction. The car is moving with a *positive velocity*, but with a *negative acceleration*. (This type of motion is exhibited by a car that skids to a stop after its brakes are applied.) The velocity and acceleration are in opposite directions. In terms of our earlier force discussion, imagine a force pulling on the car opposite to the direction it is moving: it slows down.

Each purple acceleration arrow in parts (b) and (c) of Figure 2.10 is the same length. Therefore, these diagrams represent motion of a *particle under constant acceleration*. This important analysis model will be discussed in the next section.

QUICK QUIZ 2.6 Which one of the following statements is true? **(a)** If a car is traveling eastward, its acceleration must be eastward. **(b)** If a car is slowing down, its acceleration must be negative. **(c)** A particle with constant acceleration can never stop and stay stopped.

2.7 Analysis Model: Particle Under Constant Acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. A very common and simple type of one-dimensional motion, however, is that in which the acceleration is constant. In such a case, the average acceleration $a_{x,\text{avg}}$ over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion. This situation occurs often enough that we identify it as an analysis model: the **particle under constant acceleration.** In the discussion that follows, we generate several equations that describe the motion of a particle for this model.

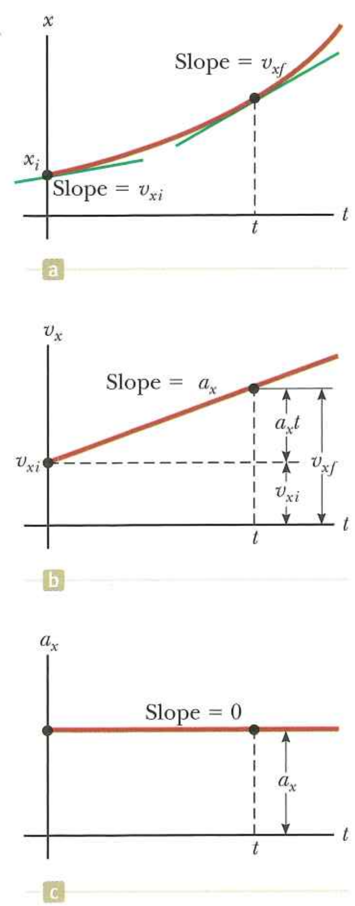


Figure 2.11 A particle under constant acceleration a_x moving along the x axis: (a) the position–time graph, (b) the velocity–time graph, and (c) the acceleration–time graph.

If we replace $a_{x,\text{avg}}$ by a_x in Equation 2.9 and take $t_i = 0$ and t_f to be any later time t, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

or

$$v_{xf} = v_{xi} + a_x t \quad (\text{for constant } a_x) \tag{2.13}$$

This powerful expression enables us to determine an object's velocity at *any* time t if we know the object's initial velocity v_{xi} and its (constant) acceleration a_x. A velocity–time graph for this constant-acceleration motion is shown in Figure 2.11b. The graph is a straight line, the slope of which is the acceleration a_x; the (constant) slope is consistent with $a_x = dv_x/dt$ being a constant. Notice that the slope is positive, which indicates a positive acceleration. If the acceleration were negative, the slope of the line in Figure 2.11b would be negative. When the acceleration is constant, the graph of acceleration versus time (Fig. 2.11c) is a straight line having a slope of zero.

Because velocity at constant acceleration varies linearly in time according to Equation 2.13, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}:

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \quad (\text{for constant } a_x) \tag{2.14}$$

Notice that this expression for average velocity applies *only* in situations in which the acceleration is constant.

We can now use Equations 2.1, 2.2, and 2.14 to obtain the position of an object as a function of time. Recalling that Δx in Equation 2.2 represents $x_f - x_i$ and recognizing that $\Delta t = t_f - t_i = t - 0 = t$, we find that

$$x_f - x_i = v_{x,\text{avg}}\, t = \tfrac{1}{2}(v_{xi} + v_{xf})t$$

Position as a function of velocity and time for the particle under constant acceleration model ▶

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \quad (\text{for constant } a_x) \tag{2.15}$$

This equation provides the final position of the particle at time t in terms of the initial and final velocities.

We can obtain another useful expression for the position of a particle under constant acceleration by substituting Equation 2.13 into Equation 2.15:

$$x_f = x_i + \tfrac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

Position as a function of time for the particle under constant acceleration model ▶

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \quad (\text{for constant } a_x) \tag{2.16}$$

This equation provides the final position of the particle at time t in terms of the initial position, the initial velocity, and the constant acceleration.

The position–time graph for motion at constant (positive) acceleration shown in Figure 2.11a is obtained from Equation 2.16. Notice that the curve is a parabola. The slope of the tangent line to this curve at $t = 0$ equals the initial velocity v_{xi}, and the slope of the tangent line at any later time t equals the velocity v_{xf} at that time.

Finally, we can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from Equation 2.13 into Equation 2.15:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^{\,2} - v_{xi}^{\,2}}{2a_x}$$

$$v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i) \quad (\text{for constant } a_x) \tag{2.17}$$

◀ Velocity as a function of position for the particle under constant acceleration model

This equation provides the final velocity in terms of the initial velocity, the constant acceleration, and the position of the particle.

For motion at *zero* acceleration, we see from Equations 2.13 and 2.16 that

$$\left.\begin{aligned} v_{xf} &= v_{xi} = v_x \\ x_f &= x_i + v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when the acceleration of a particle is zero, its velocity is constant and its position changes linearly with time. In terms of models, when the acceleration of a particle is zero, the particle under constant acceleration model reduces to the particle under constant velocity model (Section 2.3).

Equations 2.13 through 2.17 are **kinematic equations** that may be used to solve any problem involving a particle under constant acceleration in one dimension. These equations are listed together below for convenience. The choice of which equation you use in a given situation depends on what you know beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns. You should recognize that the quantities that vary during the motion are position x_f, velocity v_{xf}, and time t.

You will gain a great deal of experience in the use of these equations by solving a number of exercises and problems. Many times you will discover that more than one method can be used to obtain a solution. Remember that these equations of kinematics *cannot* be used in a situation in which the acceleration varies with time. They can be used only when the acceleration is constant.

QUICK QUIZ 2.7 In Figure 2.12, match each v_x–t graph on the top with the a_x–t graph on the bottom that best describes the motion.

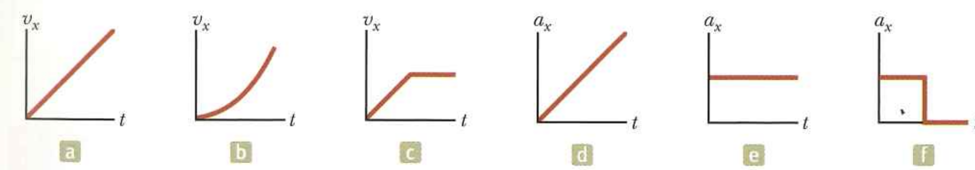


Figure 2.12 (Quick Quiz 2.7) Parts (a), (b), and (c) are v_x–t graphs of objects in one-dimensional motion. The possible accelerations of each object as a function of time are shown in scrambled order in (d), (e), and (f).

ANALYSIS MODEL Particle Under Constant Acceleration

Imagine a moving object that can be modeled as a particle. If it begins from position x_i and initial velocity v_{xi} and moves in a straight line with a constant acceleration a_x, its subsequent position and velocity are described by the following kinematic equations:

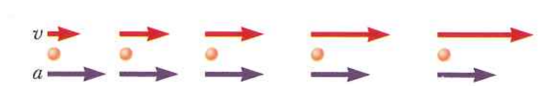
$$v_{xf} = v_{xi} + a_x t \tag{2.13}$$

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \tag{2.14}$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \tag{2.15}$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \tag{2.16}$$

$$v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i) \tag{2.17}$$



Examples

- a car accelerating at a constant rate along a straight freeway
- a dropped object in the absence of air resistance (Section 2.8)
- an object on which a constant net force acts (Chapter 5)
- a charged particle in a uniform electric field (Chapter 22)

Example 2.7 Carrier Landing

A jet lands on an aircraft carrier at a speed of 140 mi/h ($\approx$ 63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

SOLUTION

Conceptualize You might have seen movies or television shows in which a jet lands on an aircraft carrier and is brought to rest surprisingly fast by an arresting cable. A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We define our x axis as the direction of motion of the jet. Notice that we have no information about the change in position of the jet while it is slowing down.

Categorize Because the acceleration of the jet is assumed constant, we model it as a *particle under constant acceleration.*

Analyze Equation 2.13 is the only equation in the particle under constant acceleration model that does not involve position, so we use it to find the acceleration of the jet, modeled as a particle:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\text{ m/s}}{2.0\text{ s}}$$

$$= -32\text{ m/s}^2$$

(B) If the jet touches down at position $x_i = 0$, what is its final position?

SOLUTION

Use Equation 2.15 to solve for the final position: $x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(63\text{ m/s} + 0)(2.0\text{ s}) = 63\text{ m}$

Finalize Given the size of aircraft carriers, a length of 63 m seems reasonable for stopping the jet. The idea of using arresting cables to slow down landing aircraft and enable them to land safely on ships originated at about the time of World War I. The cables are still a vital part of the operation of modern aircraft carriers.

WHAT IF? Suppose the jet lands on the deck of the aircraft carrier with a speed faster than 63 m/s but has the same acceleration due to the cable as that calculated in part (A). How will that change the answer to part (B)?

Answer If the jet is traveling faster at the beginning, it will stop farther away from its starting point, so the answer to part (B) should be larger. Mathematically, we see in Equation 2.15 that if v_{xi} is larger, then x_f will be larger.

Example 2.8 Watch Out for the Speed Limit!

You are driving at a constant speed of 45.0 m/s when you pass a trooper on a motorcycle hidden behind a billboard. One second after your car passes the billboard, the trooper sets out from the billboard to catch you, accelerating at a constant rate of 3.00 m/s². How long does it take the trooper to overtake your car?

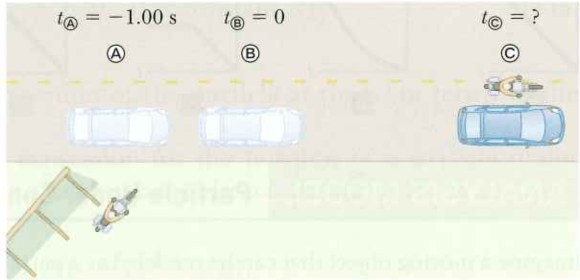


Figure 2.13 (Example 2.8) You are in a speeding car that passes a hidden trooper.

SOLUTION

Conceptualize This example represents a class of problems called *context-rich* problems. These problems involve real-world situations that one might encounter in one's daily life. These problems also involve "you" as opposed to an unspecified particle or object. With you as the character in the problem, *you* can make the connection between physics and everyday life!

Categorize A pictorial representation (Fig. 2.13) helps clarify the sequence of events. Your car is modeled as a *particle under constant velocity,* and the trooper is modeled as a *particle under constant acceleration.*

Analyze First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_{\text{Ⓑ}} = 0$ as the time the trooper begins moving. At that instant, your car has already

continued

2.8 *continued*

traveled a distance of 45.0 m from the billboard because it has traveled at a constant speed of $v_x = 45.0$ m/s for 1 s. Therefore, the initial position of your car is $x_{\text{Ⓑ}} = 45.0$ m.

Using the particle under constant velocity model, apply Equation 2.7 to give your car's position at any time t:

$$x_{\text{car}} = x_{\text{Ⓑ}} + v_{x\,\text{car}}t$$

A quick check shows that at $t = 0$, this expression gives your car's correct initial position when the trooper begins to move: $x_{\text{car}} = x_{\text{Ⓑ}} = 45.0$ m.

The trooper starts from rest at $t_{\text{Ⓑ}} = 0$ and accelerates at $a_x = 3.00$ m/s² away from the origin. Use Equation 2.16 to give her position at any time t:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_xt^2$$

$$x_{\text{trooper}} = 0 + (0)t + \tfrac{1}{2}a_xt^2 = \tfrac{1}{2}a_xt^2$$

Set the positions of your car and the trooper equal to represent the trooper overtaking your car at position Ⓒ:

$$x_{\text{trooper}} = x_{\text{car}}$$

$$\tfrac{1}{2}a_xt^2 = x_{\text{Ⓑ}} + v_{x\,\text{car}}t$$

Rearrange to give a quadratic equation:

$$\tfrac{1}{2}a_xt^2 - v_{x\,\text{car}}t - x_{\text{Ⓑ}} = 0$$

Solve the quadratic equation for the time at which the trooper catches your car (for help in solving quadratic equations, see Appendix B.2):

$$t = \frac{v_{x\,\text{car}} \pm \sqrt{v_{x\,\text{car}}^2 + 2a_xx_{\text{Ⓑ}}}}{a_x}$$

$$(1)\quad t = \frac{v_{x\,\text{car}}}{a_x} \pm \sqrt{\frac{v_{x\,\text{car}}^2}{a_x^2} + \frac{2x_{\text{Ⓑ}}}{a_x}}$$

Evaluate the solution, choosing the positive root because that is the only choice consistent with a time $t > 0$:

$$t = \frac{45.0\text{ m/s}}{3.00\text{ m/s}^2} + \sqrt{\frac{(45.0\text{ m/s})^2}{(3.00\text{ m/s}^2)^2} + \frac{2(45.0\text{ m})}{3.00\text{ m/s}^2}} = 31.0\text{ s}$$

Finalize Why didn't we choose $t = 0$ as the time at which your car passes the trooper? If we did so, we would not be able to use the particle under constant acceleration model for the trooper. Her acceleration would be zero for the first second and then 3.00 m/s² for the remaining time. By defining the time $t = 0$ as when the trooper begins moving, we can use the particle under constant acceleration model for her movement for all positive times.

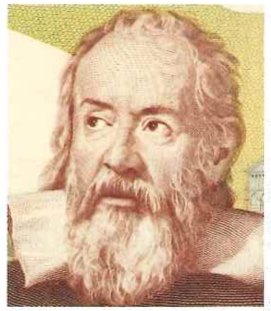
WHAT IF? What if the trooper had a more powerful motorcycle with a larger acceleration? How would that change the time at which the trooper catches your car?

Answer If the motorcycle has a larger acceleration, the trooper should catch up to your car sooner, so the answer for the time should be less than 31 s. Because all terms on the right side of Equation (1) have the acceleration a_x in the denominator, we see symbolically that increasing the acceleration will decrease the time at which the trooper catches your car.

2.8 Freely Falling Objects

It is well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity, regardless of their mass. It was not until about 1600 that this conclusion was accepted. Before that time, the teachings of the Greek philosopher Aristotle (384–322 BC) had held that heavier objects fall faster than lighter ones.

The Italian Galileo Galilei (1564–1642) originated our present-day ideas concerning falling objects. There is a legend that he demonstrated the behavior of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that he carried out this particular experiment, it is well established that Galileo performed many experiments on objects moving on inclined planes. In his experiments, he rolled balls down a slight incline and measured the distances they covered in successive time intervals. The purpose of the incline was to reduce the acceleration, which made it possible for him to make accurate measurements of the time intervals. By gradually increasing the slope of the incline,



Georgios Kollidas/Shutterstock.com

Galileo Galilei
Italian physicist and astronomer (1564–1642)
Galileo formulated the laws that govern the motion of objects in free fall and made many other significant discoveries in physics and astronomy. Galileo publicly defended Nicolaus Copernicus's assertion that the Sun is at the center of the Universe (the heliocentric system).

PITFALL PREVENTION 2.6

g and g Be sure not to confuse the italic symbol *g* for free-fall acceleration with the nonitalic symbol g used as the abbreviation for the unit gram.

PITFALL PREVENTION 2.7

The Sign of *g* Keep in mind that *g* is a *positive number.* It is tempting to substitute −9.80 m/s² for *g*, but resist the temptation. Downward gravitational acceleration is indicated explicitly by stating the acceleration as $a_y = -g$.

he was finally able to draw conclusions about freely falling objects because a freely falling ball is equivalent to a ball moving down a vertical incline.

You might want to try the following experiment. Simultaneously drop a coin and a piece of paper from the same height. The coin will always reach the ground faster. Now, crumple the paper into a tight ball and repeat the experiment. Since you've minimized the effects of air resistance, the coin and the paper will have the same motion and will hit the floor at the same time. In the idealized case, in which air resistance is absent, such motion is referred to as *free-fall* motion. If this same experiment could be conducted in a vacuum, in which air resistance is truly negligible, the paper and the coin would fall with the same acceleration even when the paper is not crumpled. On August 2, 1971, astronaut David Scott conducted such a demonstration on the Moon. He simultaneously released a hammer and a feather, and the two objects fell together to the lunar surface. This simple demonstration surely would have pleased Galileo!

When we use the expression *freely falling object,* we do not necessarily refer to an object dropped from rest. A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences an acceleration directed *downward,* regardless of its initial motion.

We shall denote the magnitude of the *free-fall acceleration,* also called the *acceleration due to gravity,* by the symbol *g*. The value of *g* decreases with increasing altitude above the Earth's surface. Furthermore, slight variations in *g* occur with changes in latitude. At the Earth's surface, the value of *g* is approximately 9.80 m/s². Unless stated otherwise, we shall use this value for *g* when performing calculations. For making quick estimates, use $g \sim 10$ m/s².

PITFALL PREVENTION 2.8

Acceleration at the Top of the Motion A common misconception is that the acceleration of a projectile at the top of its trajectory is zero. Although the velocity at the top of the motion of an object thrown upward momentarily goes to zero, *the acceleration is still that due to gravity* at this point. If the velocity and acceleration were both zero, the projectile would stay at the top.

If we neglect air resistance and assume the free-fall acceleration does not vary with altitude over short vertical distances, the motion of a freely falling object moving vertically is equivalent to the motion of a particle under constant acceleration in one dimension. Therefore, the equations developed in Section 2.7 for the particle under constant acceleration model can be applied. The only modification for freely falling objects that we need to make in these equations is to note that the motion is in the vertical direction (the *y* direction) rather than in the horizontal direction (*x*) and that the acceleration is downward and has a magnitude of 9.80 m/s². Therefore, we choose $a_y = -g = -9.80$ m/s², where the negative sign means that the acceleration of a freely falling object is downward. In Chapter 13, we shall study how to deal with variations in *g* with altitude.

QUICK QUIZ 2.8 Consider the following choices: (a) increases, (b) decreases, (c) increases and then decreases, (d) decreases and then increases, (e) remains the same. From these choices, select what happens to **(i)** the acceleration and **(ii)** the speed of a ball after it is thrown upward into the air.

Conceptual Example 2.9 The Daring Skydivers

A skydiver jumps out of a hovering helicopter. A few seconds later, another skydiver jumps out, and they both fall along the same vertical line. Ignore air resistance so that both skydivers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall?

SOLUTION

At any given instant, the speeds of the skydivers are different because one had a head start. In any time interval Δt after this instant, however, the two skydivers increase their speeds by the same amount because they have the same acceleration. Therefore, the difference in their speeds remains the same throughout the fall.

The first jumper always has a greater speed than the second. Therefore, in a given time interval, the first skydiver covers a greater distance than the second. Consequently, the separation distance between them increases.

Example 2.10 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in Figure 2.14.

(A) Using $t_{Ⓐ} = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine the time at which the stone reaches its maximum height.

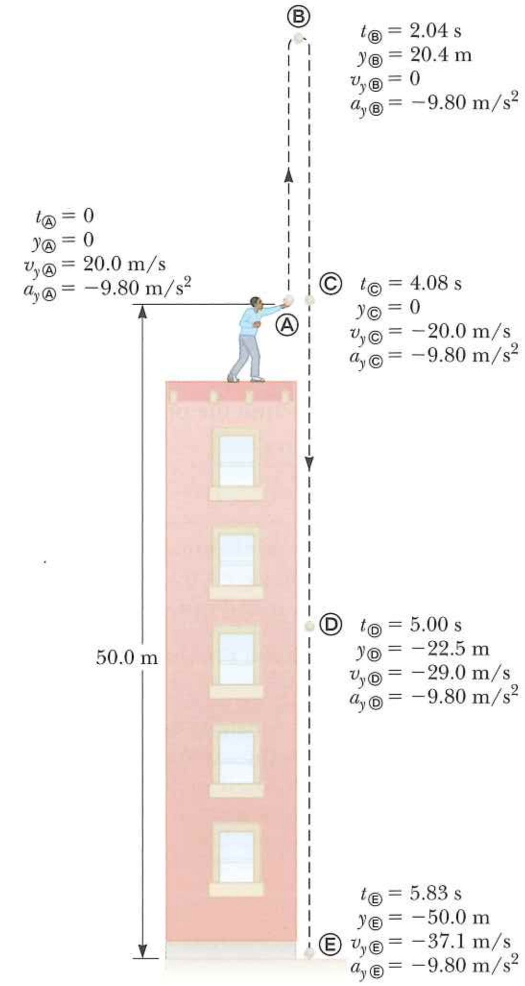


Figure 2.14 (Example 2.10) Position, velocity, and acceleration values at various times for a freely falling stone thrown initially upward with a velocity $v_{yi} = 20.0$ m/s. Many of the quantities in the labels for points in the motion of the stone are calculated in the example. Can you verify the other values that are not?

SOLUTION

Conceptualize You most likely have experience with dropping objects or throwing them upward and watching them fall, so this problem should describe a familiar experience. To simulate this situation, toss a small object upward and notice the time interval required for it to fall to the floor. Now imagine throwing that object upward from the roof of a building.

Categorize Because the stone is in free fall, it is modeled as a *particle under constant acceleration* due to gravity.

Analyze Recognize that the initial velocity is positive because the stone is launched upward. The velocity will change sign after the stone reaches its highest point, but the acceleration of the stone will *always* be downward so that it will always have a negative value. Choose an initial point just after the stone leaves the person's hand and a final point at the top of its flight.

Use Equation 2.13 to calculate the time at which the stone reaches its maximum height:

$$v_{yf} = v_{yi} + a_y t \rightarrow t = \frac{v_{yf} - v_{yi}}{a_y} = \frac{v_{yⒷ} - v_{yⒶ}}{-g}$$

Substitute numerical values, recognizing that $v = 0$ at point Ⓑ:

$$t = t_{Ⓑ} = \frac{0 - 20.0\text{ m/s}}{-9.80\text{ m/s}^2} = 2.04\text{ s}$$

(B) Find the maximum height of the stone.

SOLUTION

As in part (A), choose the initial and final points at the beginning and the end of the upward flight.

Set $y_{Ⓐ} = 0$ and substitute the time from part (A) into Equation 2.16 to find the maximum height:

$$y_{max} = y_{Ⓑ} = y_{Ⓐ} + v_{xⒶ}t + \tfrac{1}{2}a_y t^2$$

$$y_{Ⓑ} = 0 + (20.0\text{ m/s})(2.04\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(2.04\text{ s})^2 = 20.4\text{ m}$$

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

SOLUTION

Choose the initial point where the stone is launched and the final point when it passes this position coming down.

Substitute known values into Equation 2.17:

$$v_{yⒸ}^2 = v_{yⒶ}^2 + 2a_y(y_{Ⓒ} - y_{Ⓐ})$$

$$v_{yⒸ}^2 = (20.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(0 - 0) = 400\text{ m}^2/\text{s}^2$$

$$v_{yⒸ} = -20.0\text{ m/s}$$

continued

2.10 continued

When taking the square root, we could choose either a positive or a negative root. We choose the negative root because we know that the stone is moving downward at point Ⓒ. The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but is opposite in direction.

(D) Find the velocity and position of the stone at $t = 5.00$ s.

SOLUTION

Choose the initial point just after the throw and the final point 5.00 s later.

Calculate the velocity at Ⓓ from Equation 2.13:

$$v_{y\text{Ⓓ}} = v_{y\text{Ⓐ}} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s}) = -29.0 \text{ m/s}$$

Use Equation 2.16 to find the position of the stone at $t_{\text{Ⓓ}} = 5.00$ s:

$$y_{\text{Ⓓ}} = y_{\text{Ⓐ}} + v_{y\text{Ⓐ}} t + \tfrac{1}{2} a_y t^2$$
$$= 0 + (20.0 \text{ m/s})(5.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s})^2$$
$$= -22.5 \text{ m}$$

Finalize The choice of the time defined as $t = 0$ is arbitrary and up to you to select as the problem solver. As an example of this arbitrariness, choose $t = 0$ as the time at which the stone is at the highest point in its motion. Then solve parts (C) and (D) again using this new initial instant and notice that your answers are the same as those above.

WHAT IF? What if the throw were from 30.0 m above the ground instead of 50.0 m? Which answers in parts (A) to (D) would change?

Answer None of the answers would change. All the motion takes place in the air during the first 5.00 s. (Notice that even for a throw from 30.0 m, the stone is above the ground at $t = 5.00$ s.) Therefore, the height from which the stone is thrown is not an issue. Mathematically, if we look back over our calculations, we see that we never entered the height from which the stone is thrown into any equation.

2.9 Kinematic Equations Derived from Calculus

PITFALL PREVENTION 2.9
Previous Experience with Integration This section assumes the reader is familiar with the techniques of integral calculus. If you have not yet studied integration in your calculus course, you should skip this section or cover it after you become familiar with integration.

The velocity of a particle moving in a straight line can be determined as the derivative of the position with respect to time. It is also possible to find the position of a particle if its velocity is known as a function of time. In calculus, the procedure used to perform this task is referred to either as *integration* or as finding the *antiderivative*.

Suppose the v_x–t graph for a particle moving along the x axis is as shown in Figure 2.15. Let us divide the time interval $t_f - t_i$ into many small intervals, each of duration Δt_n. From the definition of average velocity, we see that the displacement of the particle during any small interval, such as the one shaded in Figure 2.15, is given by $\Delta x_n = v_{xn,\text{avg}}\,\Delta t_n$, where $v_{xn,\text{avg}}$ is the average velocity in that interval. Therefore, the displacement during this small interval is simply the area of the shaded rectangle in Figure 2.15. The total displacement for the interval $t_f - t_i$ is the sum of the areas of all the rectangles from t_i to t_f:

$$\Delta x = \sum_n v_{xn,\text{avg}}\,\Delta t_n$$

where the symbol Σ (uppercase Greek sigma) signifies a sum over all terms, that is, over all values of n. Now, as the intervals are made smaller and smaller, the number of terms in the sum increases and the sum approaches a value equal to the area under the curve in the velocity–time graph. Therefore, in the limit $n \to \infty$, or $\Delta t_n \to 0$, the displacement is

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn,\text{avg}}\,\Delta t_n \quad (2.18)$$

The limit of the sum shown in Equation 2.18 is called a **definite integral** and so the displacement of the particle can be written as

Definite integral ▶

$$\Delta x = \int_{t_i}^{t_f} v_x(t)\,dt \quad (2.19)$$

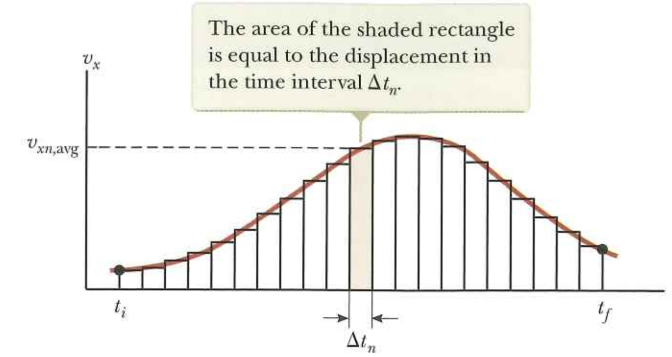


Figure 2.15 Velocity versus time for a particle moving along the x axis. The total area under the curve is the total displacement of the particle.

where $v_x(t)$ denotes the velocity at any time t. If the explicit functional form of $v_x(t)$ is known and the limits are given, the integral can be evaluated.

PITFALL PREVENTION 2.10
Integration is an Area If this discussion of integration is confusing to you, just remember that the integral of a function is simply the area between the function and the x axis between the limits of integration. If the function has a simple shape, the area can be easily calculated without integration. For example, if the function is a constant, so that its graph is a horizontal line, the area is just that of the rectangle between the line and the x axis!

Kinematic Equations

We now use the defining equations for acceleration and velocity to derive two of our kinematic equations, Equations 2.13 and 2.16.

The defining equation for acceleration (Eq. 2.10),

$$a_x = \frac{dv_x}{dt}$$

may be written as $dv_x = a_x\,dt$ or, in terms of an integral (or antiderivative), as

$$v_{xf} - v_{xi} = \int_0^t a_x\,dt$$

For the special case in which the acceleration is constant, a_x can be removed from the integral to give

$$v_{xf} - v_{xi} = a_x \int_0^t dt = a_x(t - 0) = a_x t \quad (2.20)$$

which is Equation 2.13 in the particle under constant acceleration model.

Now let us consider the defining equation for velocity (Eq. 2.5):

$$v_x = \frac{dx}{dt}$$

We can write this equation as $dx = v_x\,dt$ or in integral form as

$$x_f - x_i = \int_0^t v_x\,dt$$

Because $v_x = v_{xf} = v_{xi} + a_x t$, this expression becomes

$$x_f - x_i = \int_0^t (v_{xi} + a_x t)\,dt = \int_0^t v_{xi}\,dt + a_x \int_0^t t\,dt = v_{xi}(t - 0) + a_x\left(\frac{t^2}{2} - 0\right)$$

$$x_f - x_i = v_{xi}t + \tfrac{1}{2}a_x t^2$$

which is Equation 2.16 in the particle under constant acceleration model.

Summary

Definitions

When a particle moves along the x axis from some initial position x_i to some final position x_f, its **displacement** is

$$\Delta x \equiv x_f - x_i \quad (2.1)$$

The **average velocity** of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurs:

$$v_{x,\text{avg}} \equiv \frac{\Delta x}{\Delta t} \quad (2.2)$$

The **average speed** of a particle is equal to the ratio of the total distance it travels to the total time interval during which it travels that distance:

$$v_{\text{avg}} \equiv \frac{d}{\Delta t} \quad (2.3)$$

The **instantaneous velocity** of a particle is defined as the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero. By definition, this limit equals the derivative of x with respect to t, or the time rate of change of the position:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (2.5)$$

The **instantaneous speed** of a particle is equal to the magnitude of its instantaneous velocity.

The **average acceleration** of a particle is defined as the ratio of the change in its velocity Δv_x divided by the time interval Δt during which that change occurs:

$$a_{x,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad (2.9)$$

The **instantaneous acceleration** is equal to the limit of the ratio $\Delta v_x/\Delta t$ as Δt approaches 0. By definition, this limit equals the derivative of v_x with respect to t, or the time rate of change of the velocity:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad (2.10)$$

Concepts and Principles

When an object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down. Remembering that $F_x \propto a_x$ is a useful way to identify the direction of the acceleration by associating it with a force.

An object falling freely in the presence of the Earth's gravity experiences free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, the free-fall acceleration $a_y = -g$ is constant over the range of motion, where g is equal to 9.80 m/s^2.

Complicated problems are best approached in an organized manner. Recall and apply the *Conceptualize, Categorize, Analyze,* and *Finalize* steps of the **Analysis Model Approach to Problem Solving** when you need them.

An important aid to problem solving is the use of **analysis models.** Analysis models are situations that we have seen in previous problems. Each analysis model has one or more equations associated with it. When solving a new problem, identify the analysis model that corresponds to the problem. The model will tell you which equations to use. The first three analysis models introduced in this chapter are summarized below.

Analysis Models for Problem Solving

Particle Under Constant Velocity. If a particle moves in a straight line with a constant speed v_x, its constant velocity is given by

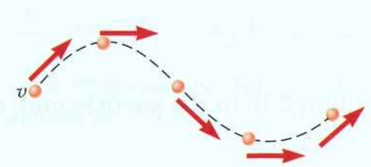
$$v_x = \frac{\Delta x}{\Delta t} \quad (2.6)$$

and its position is given by

$$x_f = x_i + v_x t \quad (2.7)$$

Particle Under Constant Speed. If a particle moves a distance d along a curved or straight path with a constant speed, its constant speed is given by

$$v = \frac{d}{\Delta t} \quad (2.8)$$



Particle Under Constant Acceleration. If a particle moves in a straight line with a constant acceleration a_x, its motion is described by the kinematic equations:

$$v_{xf} = v_{xi} + a_x t \quad (2.13)$$

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \quad (2.14)$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \quad (2.15)$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \quad (2.16)$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \quad (2.17)$$

Think–Pair–Share

See the Preface for an explanation of the icons used in this problems set. For additional assessment items for this section, go to WEBASSIGN From Cengage

1. You are at a carnival playing the "Strike-the-Bell" game, as shown in Figure TP2.1. The goal is to hit the end of the lever with a hammer, sending a hard object upward along the frictionless vertical track so as to strike a bell at the top. Showing off your control for the crowd, you hit the lever several times in a row in such a way that the hard object rises to a height h = 4.50 m and just touches the bell, which makes a gentle ringing sound. Now, to really impress the crowd, you swing the hammer with a mighty motion, hit the lever, and project the object upward with twice the initial speed of your previous demonstrations. Unbeknownst to you, on the previous demonstration, the bell came loose and slipped off to the side, so that, on this demonstration, the object bypasses the bell and is projected straight up into the air. What is the total time interval between when the object begins its upward motion and then later lands on the ground beside the apparatus?

Figure TP2.1

2. Your group is at the top of a cliff of height h = 75.0 m. At the bottom of the cliff is a pool of water. You split the group in two. The first half of the group volunteers a member to drop a rock from rest so that it falls straight downward and makes a splash in the water. The second half of the group volunteers a member to, after some time interval has passed since the first rock was dropped, throw a second rock straight downward so that both rocks arrive at the water at the same time. You test the performance by listening for a single splash made by the rocks simultaneously hitting the water. (a) If the second rock is thrown 1.00 s after the first rock is released, with what speed must the second rock be thrown? (b) If the fastest anyone in your group can throw the rock is 40.0 m/s, what is the longest time interval that can pass between the release of the rocks so that a single splash is heard? (c) If there is no limit as to how fast the rock can be thrown, what is the longest time interval that can pass between the release of the rocks so that a single splash is heard?

3. **ACTIVITY** Have your partner hold a ruler vertically with the zero end at the bottom. Place your open finger and thumb at the zero position. Without warning, your partner should release the ruler and you should catch it as soon as you see it moving. From the position of your finger on the ruler, determine your reaction time. Repeat the experiment a number of times to estimate the uncertainty in your reaction time. Have each member of your group catch the ruler and compare your reaction times.

4. **ACTIVITY** The Acela is an electric train on the Washington–New York–Boston run, carrying passengers at speeds as high as 170 mi/h. A velocity–time graph for the Acela is shown in Figure TP2.4. (a) Describe the train's motion in each successive time interval. (b) Find the train's peak positive acceleration in the motion graphed. (c) Find the train's displacement in miles between t = 0 and t = 200 s.

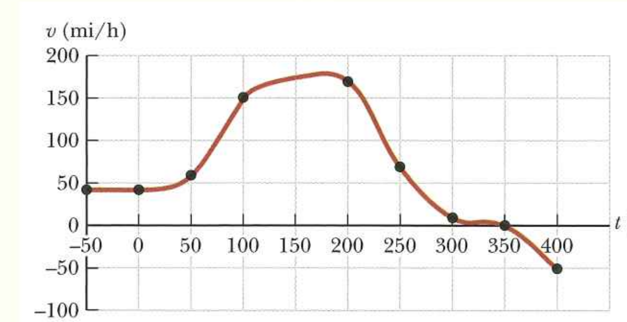


Figure TP2.4 Velocity–time graph for the Acela.

Problems

See the Preface for an explanation of the icons used in this problems set. For additional assessment items for this section, go to WEBASSIGN From Cengage

SECTION 2.1 Position, Velocity, and Speed

1. BIO The speed of a nerve impulse in the human body is about 100 m/s. If you accidentally stub your toe in the dark, estimate the time it takes the nerve impulse to travel to your brain.

2. V A particle moves according to the equation $x = 10t^2$, where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.00 s to 3.00 s. (b) Find the average velocity for the time interval from 2.00 to 2.10 s.

3. The position of a pinewood derby car was observed at various times; the results are summarized in the following table. Find the average velocity of the car for (a) the first second, (b) the last 3 s, and (c) the entire period of observation.

t (s)	0	1.0	2.0	3.0	4.0	5.0
x (m)	0	2.3	9.2	20.7	36.8	57.5

SECTION 2.2 Instantaneous Velocity and Speed

4. S An athlete leaves one end of a pool of length L at $t = 0$ and arrives at the other end at time t_1. She swims back and arrives at the starting position at time t_2. If she is swimming initially in the positive x direction, determine her average velocities symbolically in (a) the first half of the swim, (b) the second half of the swim, and (c) the round trip. (d) What is her average speed for the round trip?

5. A position–time graph for a particle moving along the x axis is shown in Figure P2.5. (a) Find the average velocity in the time interval $t = 1.50$ s to $t = 4.00$ s. (b) Determine the instantaneous velocity at $t = 2.00$ s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?

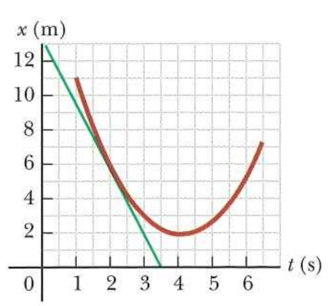


Figure P2.5

SECTION 2.3 Analysis Model: Particle Under Constant Velocity

6. AMT A car travels along a straight line at a constant speed of 60.0 mi/h for a distance d and then another distance d in the same direction at another constant speed. The average velocity for the entire trip is 30.0 mi/h. (a) What is the constant speed with which the car moved during the second distance d? (b) **What If?** Suppose the second distance d were traveled in the opposite direction; you forgot something and had to return home at the same constant speed as found in part (a). What is the average velocity for this trip? (c) What is the average speed for this new trip?

7. T A person takes a trip, driving with a constant speed of 89.5 km/h, except for a 22.0-min rest stop. If the person's average speed is 77.8 km/h, (a) how much time is spent on the trip and (b) how far does the person travel?

SECTION 2.5 Acceleration

8. A child rolls a marble on a bent track that is 100 cm long as shown in Figure P2.8. We use x to represent the position of the marble along the track. On the horizontal sections from $x = 0$ to $x = 20$ cm and from $x = 40$ cm to $x = 60$ cm, the marble rolls with constant speed. On the sloping sections, the marble's speed changes steadily. At the places where the slope changes, the marble stays on the track and does not undergo any sudden changes in speed. The child gives the marble some initial speed at $x = 0$ and $t = 0$ and then watches it roll to $x = 90$ cm, where it turns around, eventually returning to $x = 0$ with the same speed with which the child released it. Prepare graphs of x versus t, v_x versus t, and a_x versus t, vertically aligned with their time axes identical, to show the motion of the marble. You will not be able to place numbers other than zero on the horizontal axis or on the velocity or acceleration axes, but show the correct graph shapes.

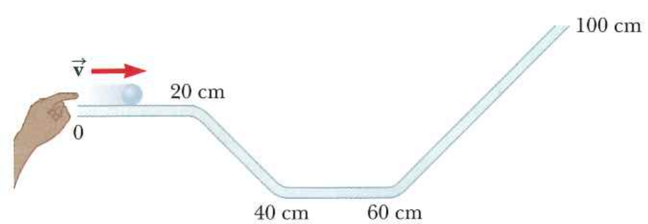


Figure P2.8

9. Figure P2.9 shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval $t = 0$ to $t = 6.00$ s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs.

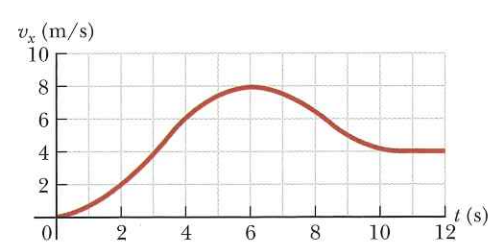


Figure P2.9

10. (a) Use the data in Problem 3 to construct a smooth graph of position versus time. (b) By constructing tangents to the $x(t)$ curve, find the instantaneous velocity of the car at several instants. (c) Plot the instantaneous velocity versus time and, from this information, determine the average acceleration of the car. (d) What was the initial velocity of the car?

11. V A particle starts from rest and accelerates as shown in Figure P2.11. Determine (a) the particle's speed at $t = 10.0$ s and at $t = 20.0$ s, and (b) the distance traveled in the first 20.0 s.

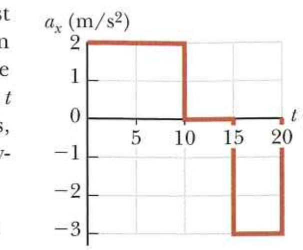


Figure P2.11

SECTION 2.6 Motion Diagrams

12. Q|C Draw motion diagrams for (a) an object moving to the right at constant speed, (b) an object moving to the right and speeding up at a constant rate, (c) an object moving to the right and slowing down at a constant rate, (d) an object moving to the left and speeding up at a constant rate, and (e) an object moving to the left and slowing down at a constant rate. (f) How would your drawings change if the changes in speed were not uniform, that is, if the speed were not changing at a constant rate?

13. Q|C Each of the strobe photographs (a), (b), and (c) in Figure P2.13 was taken of a single disk moving toward the right, which we take as the positive direction. Within each photograph the time interval between images is constant. For each photograph, prepare graphs of x versus t, v_x versus t, and a_x versus t, vertically aligned with their time axes identical, to show the motion of the disk. You will not be able to place numbers other than zero on the axes, but show the correct shapes for the graph lines.

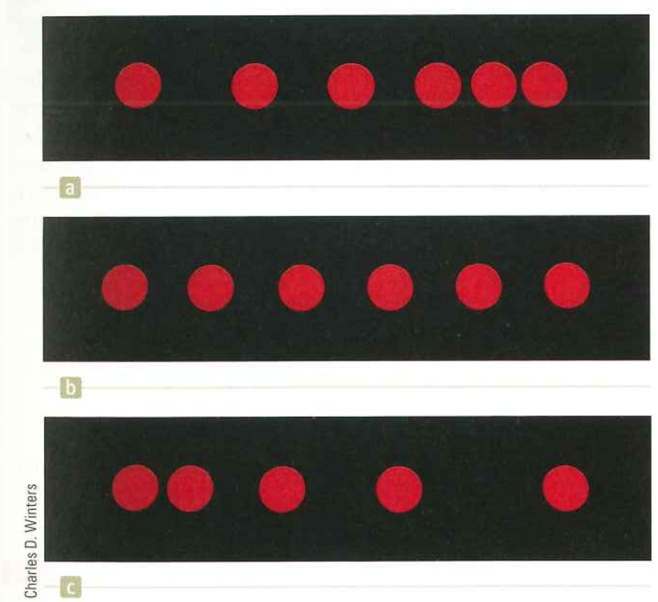


Figure P2.13

SECTION 2.7 Analysis Model: Particle Under Constant Acceleration

14. An electron in a cathode-ray tube accelerates uniformly from 2.00×10^4 m/s to 6.00×10^6 m/s over 1.50 cm. (a) In what time interval does the electron travel this 1.50 cm? (b) What is its acceleration?

15. Q|C A parcel of air moving in a straight tube with a constant acceleration of -4.00 m/s² has a velocity of 13.0 m/s at 10:05:00 a.m. (a) What is its velocity at 10:05:01 a.m.? (b) At 10:05:04 a.m.? (c) At 10:04:59 a.m.? (d) Describe the shape of a graph of velocity versus time for this parcel of air. (e) Argue for or against the following statement: "Knowing the single value of an object's constant acceleration is like knowing a whole list of values for its velocity."

16. T Q|C In Example 2.7, we investigated a jet landing on an aircraft carrier. In a later maneuver, the jet comes in for a landing on solid ground with a speed of 100 m/s, and its acceleration can have a maximum magnitude of 5.00 m/s² as it comes to rest. (a) From the instant the jet touches the runway, what is the minimum time interval needed before it can come to rest? (b) Can this jet land at a small tropical island airport where the runway is 0.800 km long? (c) Explain your answer.

17. T An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is its acceleration?

18. Solve Example 2.8 by a graphical method. On the same graph, plot position versus time for the car and the trooper. From the intersection of the two curves, read the time at which the trooper overtakes the car.

19. Q|C A glider of length ℓ moves through a stationary photogate on an air track. A photogate (Fig. P2.19) is a device that measures the time interval Δt_d during which the glider blocks a beam of infrared light passing across the photogate. The ratio $v_d = \ell/\Delta t_d$ is the average velocity of the glider over this part of its motion. Suppose the glider moves with constant acceleration. (a) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in space. (b) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in time.

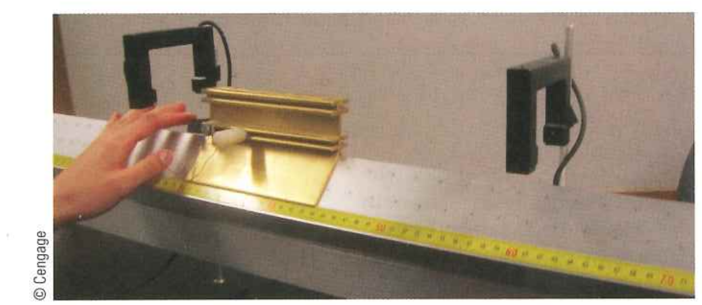


Figure P2.19 Problems 19 and 21.

20. *Why is the following situation impossible?* Starting from rest, a charging rhinoceros moves 50.0 m in a straight line in 10.0 s. Her acceleration is constant during the entire motion, and her final speed is 8.00 m/s.

21. Q|C A glider of length 12.4 cm moves on an air track with constant acceleration (Fig P2.19). A time interval of 0.628 s elapses between the moment when its front end passes a fixed point Ⓐ along the track and the moment when its back end passes this point. Next, a time interval of 1.39 s elapses between the moment when the back end of the glider passes the point Ⓐ and the moment when the front end of the glider passes a second point Ⓑ farther down the track. After that, an additional 0.431 s elapses until the back end of the glider passes point Ⓑ. (a) Find the average speed of the glider as it passes point Ⓐ. (b) Find the acceleration of the glider. (c) Explain how you can compute the acceleration without knowing the distance between points Ⓐ and Ⓑ.

22. S In the particle under constant acceleration model, we identify the variables and parameters v_{xi}, v_{xf}, a_x, t, and

$x_f - x_i$. Of the equations in the model, Equations 2.13–2.17, the first does not involve $x_f - x_i$, the second and third do not contain a_x, the fourth omits v_{xf}, and the last leaves out t. So, to complete the set, there should be an equation *not* involving v_{xi}. Derive it from the others.

23. Q|C At $t = 0$, one toy car is set rolling on a straight track with initial position 15.0 cm, initial velocity −3.50 cm/s, and constant acceleration 2.40 cm/s². At the same moment, another toy car is set rolling on an adjacent track with initial position 10.0 cm, initial velocity +5.50 cm/s, and constant acceleration zero. (a) At what time, if any, do the two cars have equal speeds? (b) What are their speeds at that time? (c) At what time(s), if any, do the cars pass each other? (d) What are their locations at that time? (e) Explain the difference between question (a) and question (c) as clearly as possible.

24. CR You are observing the poles along the side of the road as described in the opening storyline of the chapter. You have already stopped and measured the distance between adjacent poles as 40.0 m. You are now driving again and have activated your smartphone stopwatch. You start the stopwatch at $t = 0$ as you pass pole #1. At pole #2, the stopwatch reads 10.0 s. At pole #3, the stopwatch reads 25.0 s. Your friend tells you that he was pressing the brake and slowing down the car uniformly during the entire time interval from pole #1 to pole #3. (a) What was the acceleration of the car between poles #1 and #3? (b) What was the velocity of the car at pole #1? (c) If the motion of the car continues as described, what is the number of the *last* pole passed before the car comes to rest?

SECTION 2.8 Freely Falling Objects

Note: In all problems in this section, ignore the effects of air resistance.

25. *Why is the following situation impossible?* Emily challenges David to catch a $1 bill as follows. She holds the bill vertically as shown in Figure P2.25, with the center of the bill between but not touching David's index finger and thumb. Without warning, Emily releases the bill. David catches the bill without moving his hand downward. David's reaction time is equal to the average human reaction time.

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Figure P2.25

26. Q|C An attacker at the base of a castle wall 3.65 m high throws a rock straight up with speed 7.40 m/s from a height of 1.55 m above the ground. (a) Will the rock reach the top of the wall? (b) If so, what is its speed at the top? If not, what initial speed must it have to reach the top? (c) Find the change in speed of a rock thrown straight down from the top of the wall at an initial speed of 7.40 m/s and moving between the same two points. (d) Does the change in speed of the downward-moving rock agree with the magnitude of the speed change of the rock moving upward between the same elevations? (e) Explain physically why it does or does not agree.

27. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. At $t = 2.00$ s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

28. T A ball is thrown upward from the ground with an initial speed of 25 m/s; at the same instant, another ball is dropped from a building 15 m high. After how long will the balls be at the same height above the ground?

29. T A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The second student catches the keys 1.50 s later. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

30. S At time $t = 0$, a student throws a set of keys vertically upward to her sorority sister, who is in a window at distance h above. The second student catches the keys at time t. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

31. CR You have been hired by the prosecuting attorney as an expert witness in a robbery case. The defendant is accused of stealing an expensive and massive diamond ring in its box from a jewelry store. A witness to the alleged crime testified that she saw the defendant run from the store, stop next to an apartment building, and throw the box straight upward to an accomplice leaning out a fourth-floor window. When captured, the defendant did not have the stolen box with him and claimed innocence. When the witness testified in court about the defendant's throwing of the box to an accomplice, the defending attorney argued that it would be impossible to throw the box upward that high to reach the window in question. The bottom of the window is 19.0 m above the sidewalk. You have set up a demonstration in which the defendant was asked by the judge to throw a baseball horizontally as fast as he could and a radar device was used to determine that he can throw the ball at 20 m/s. (a) What testimony can you provide about the ability of the defendant to throw the box to the window in question? (b) What argument might the defense attorney make about the process used to develop your expert testimony? What might be your counter argument? Ignore any effects of air resistance on the box.

SECTION 2.9 Kinematic Equations Derived from Calculus

32. A student drives a moped along a straight road as described by the velocity–time graph in Figure P2.32. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the velocity–time graph, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6.00$ s? (d) Find the position (relative to the starting point) at $t = 6.00$ s. (e) What is the moped's final position at $t = 9.00$ s?

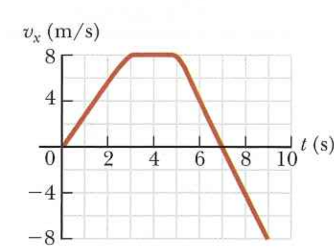

Figure P2.32

33. S Automotive engineers refer to the time rate of change of acceleration as the "jerk." Assume an object moves in one dimension such that its jerk J is constant. (a) Determine expressions for its acceleration $a_x(t)$, velocity $v_x(t)$, and position $x(t)$, given that its initial acceleration, velocity, and position are a_{xi}, v_{xi}, and x_i, respectively. (b) Show that $a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})$.

ADDITIONAL PROBLEMS

34. Q|C S In Figure 2.11b, the area under the velocity–time graph and between the vertical axis and time t (vertical dashed line) represents the displacement. As shown, this area consists of a rectangle and a triangle. (a) Compute their areas. (b) Explain how the sum of the two areas compares with the expression on the right-hand side of Equation 2.16.

35. BIO The froghopper *Philaenus spumarius* is supposedly the best jumper in the animal kingdom. To start a jump, this insect can accelerate at 4.00 km/s² over a distance of 2.00 mm as it straightens its specially adapted "jumping legs." Assume the acceleration is constant. (a) Find the upward velocity with which the insect takes off. (b) In what time interval does it reach this velocity? (c) How high would the insect jump if air resistance were negligible? The actual height it reaches is about 70 cm, so air resistance must be a noticeable force on the leaping froghopper.

36. A woman is reported to have fallen 144 ft from the 17th floor of a building, landing on a metal ventilator box that she crushed to a depth of 18.0 in. She suffered only minor injuries. Ignoring air resistance, calculate (a) the speed of the woman just before she collided with the ventilator and (b) her average acceleration while in contact with the box. (c) Modeling her acceleration as constant, calculate the time interval it took to crush the box.

37. Q|C S At $t = 0$, one athlete in a race running on a long, straight track with a constant speed v_1 is a distance d_1 behind a second athlete running with a constant speed v_2. (a) Under what circumstances is the first athlete able to overtake the second athlete? (b) Find the time t at which the first athlete overtakes the second athlete, in terms of d_1, v_1, and v_2. (c) At what minimum distance d_2 from the leading athlete must the finish line be located so that the trailing athlete can at least tie for first place? Express d_2 in terms of d_1, v_1, and v_2 by using the result of part (b).

38. *Why is the following situation impossible?* A freight train is lumbering along at a constant speed of 16.0 m/s. Behind the freight train on the same track is a passenger train traveling in the same direction at 40.0 m/s. When the front of the passenger train is 58.5 m from the back of the freight train, the engineer on the passenger train recognizes the danger and hits the brakes of his train, causing the train to move with acceleration −3.00 m/s². Because of the engineer's action, the trains do not collide.

39. AMT T Hannah tests her new sports car by racing with Sam, an experienced racer. Both start from rest, but Hannah leaves the starting line 1.00 s after Sam does. Sam moves with a constant acceleration of 3.50 m/s², while Hannah maintains an acceleration of 4.90 m/s². Find (a) the time at which Hannah overtakes Sam, (b) the distance she travels before she catches him, and (c) the speeds of both cars at the instant Hannah overtakes Sam.

40. Q|C S Two objects, A and B, are connected by hinges to a rigid rod that has a length L. The objects slide along perpendicular guide rails as shown in Figure P2.40. Assume object A slides to the left with a constant speed v. (a) Find the velocity v_B of object B as a function of the angle θ. (b) Describe v_B relative to v. Is v_B always smaller than v, larger than v, or the same as v, or does it have some other relationship?

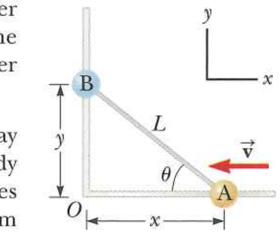

Figure P2.40

41. Lisa rushes down onto a subway platform to find her train already departing. She stops and watches the cars go by. Each car is 8.60 m long. The first moves past her in 1.50 s and the second in 1.10 s. Find the constant acceleration of the train.

CHALLENGE PROBLEMS

42. Two thin rods are fastened to the inside of a circular ring as shown in Figure P2.42. One rod of length D is vertical, and the other of length L makes an angle θ with the horizontal. The two rods and the ring lie in a vertical plane. Two small beads are free to slide without friction along the rods. (a) If the two beads are released from rest simultaneously from the positions shown, use your intuition and guess which bead reaches the bottom first. (b) Find an expression for the time interval required for the red bead to fall from point Ⓐ to point Ⓒ in terms of g and D. (c) Find an expression for the time interval required for the blue bead to slide from point Ⓑ to point Ⓒ in terms of g, L, and θ. (d) Show that the two time intervals found in parts (b) and (c) are equal. *Hint:* What is the angle between the chords of the circle Ⓐ Ⓑ and Ⓑ Ⓒ? (e) Do these results surprise you? Was your intuitive guess in part (a) correct? This problem was inspired by an article by Thomas B. Greenslade, Jr., "Galileo's Paradox," *Phys. Teach.* **46**, 294 (May 2008).

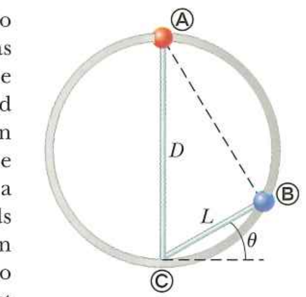

Figure P2.42

43. In a women's 100-m race, accelerating uniformly, Laura takes 2.00 s and Healan 3.00 s to attain their maximum speeds, which they each maintain for the rest of the race. They cross the finish line simultaneously, both setting a world record of 10.4 s. (a) What is the acceleration of each sprinter? (b) What are their respective maximum speeds? (c) Which sprinter is ahead at the 6.00-s mark, and by how much? (d) What is the maximum distance by which Healan is behind Laura, and at what time does that occur?

44. **Review.** You are sitting in your car at rest at a traffic light with a bicyclist at rest next to you in the adjoining bicycle lane. As soon as the traffic light turns green, your car speeds up from rest to 50.0 mi/h with constant acceleration 9.00 mi/h/s and thereafter moves with a constant speed of 50.0 mi/h. At the same time, the cyclist speeds up from rest to 20.0 mi/h with constant acceleration 13.0 mi/h/s and thereafter moves with a constant speed of 20.0 mi/h. (a) For what time interval after the light turned green is the bicycle ahead of your car? (b) What is the maximum distance by which the bicycle leads your car during this time interval?